

## Charmless $\bar{B}_s \rightarrow VV$ decays in QCD factorization

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The two body charmless decays of  $B_s$  mesons to light vector mesons are analyzed within the framework of QCD factorization. This approach implies that the nonfactorizable corrections to different helicity amplitudes are not the same. The effective parameters  $a_i^h$  for helicity  $h=0, +, -$  states receive different nonfactorizable contributions and hence are helicity dependent, contrary to the naive factorization approach where  $a_i^h$  are universal and polarization independent. The branching ratios for  $\bar{B}_s \rightarrow VV$  decays are calculated and we find that the branching ratios of some channels are of the order of  $10^{-5}$ , which are measurable at future experiments. The transverse to total decay rate  $\Gamma_T/\Gamma$  is also evaluated and found to be very small for most decay modes; so, in charmless  $\bar{B}_s \rightarrow VV$  decays, both light vector mesons tend to have zero helicity.

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### I. INTRODUCTION

The charmless two-body  $B$  decays play a crucial role in determining the flavor parameters, especially the Cabibbo-Kobayashi-Maskawa (CKM) angles  $\alpha$ ,  $\beta$ , and  $\gamma$ . With precise measurements of these parameters, we can explore  $CP$  violation which is described by the phase of the CKM matrix in the standard model (SM). Recently there has been remarkable progress in the study of exclusive charmless  $B$  decays, both experimentally and theoretically. On the experimental aspect, many two-body nonleptonic charmless  $B$  decays have been observed by CLEO and  $B$  factories at KEK and SLAC [1,2] and more  $B$  decay channels will be measured with great precision in the near future. With the accumulation of data, the SM can be tested in more detail. Theoretically, several novel methods have also been proposed to study the nonfactorizable effects in the hadronic matrix elements, such as QCD factorization (QCDF) [3], the perturbative QCD (PQCD) method [4], and so on. Intensive investigations on hadronic charmless two-body  $B_{u,d}$  decays with these methods have been studied in detail [5–9].

The extension of QCDF from  $B_{u,d}$  decays to  $B_s$  decays has also been carried out by several authors [10,11]. In principle, the physics of the  $B_s$  two-body hadronic decays is very similar to that for the  $B_d$  meson, except that the spectator  $d$  quark is replaced by the  $s$  quark. However, the problem is that the  $B_s$  meson oscillates at a high frequency, and nonleptonic  $B_s$  decays have still remained elusive from observation. Unlike the  $B_{u,d}$  mesons, the heavier  $B_s$  meson cannot be studied at the  $B$  factories operating at the  $Y(4s)$  resonance. But it is believed that in the forthcoming hadron colliders such as the Collider Detector at Fermilab (CDF), D0, DESY  $ep$  collider HERA-B, BTeV, and CERN Large Hadron Collider (LHCb),  $CP$  violation in the  $B_s$  system can be observed with high accuracy. This makes the search for  $CP$  violation in the  $B_s$  system decays very interesting.

In the papers [11,12], the authors have studied systematically the  $B_s \rightarrow PP, PV$  decays (here  $P, V$  denote pseudoscalar

and vector mesons, respectively) with QCD factorization, and intensive phenomenological analysis has been made. Since the  $B \rightarrow VV$  modes reveal dynamics of exclusive  $B$  meson decays more than the  $B \rightarrow PP$  and  $PV$  modes through the measurement of the magnitudes and the phases of various helicity amplitudes, in the present work we plan to make a detailed study of  $\bar{B}_s \rightarrow VV$  decays within the same framework of QCD factorization. We find that, contrary to the generalized factorization approach, nonfactorizable corrections to each helicity amplitude are not the same; the effective parameters  $a_i^h$  vary for different helicity amplitudes. The transverse to total decay rate  $\Gamma_T/\Gamma$  is very small for most decay modes, so in the heavy quark limit, both light vector mesons in charmless  $\bar{B}_s \rightarrow VV$  decays tend to have zero helicity. Branching ratios for some decay modes are found of order  $10^{-5}$ , which could be measured at LHCb.

This paper is organized as follows. In Sec. II, we outline the necessary ingredients of the QCD factorization approach for describing the  $\bar{B}_s \rightarrow VV$  decays and calculate the effective parameters  $a_i^h$ . Input parameters, numerical calculations, and results are presented in Sec. III. Finally we conclude with a summary in Sec. IV. The amplitudes for charmless two-body  $\bar{B}_s \rightarrow VV$  decays are given in the Appendix.

### II. $\bar{B}_s \rightarrow VV$ IN QCD FACTORIZATION APPROACH

#### A. The effective Hamiltonian

Using the operator product expansion and renormalization group equation, the low energy effective Hamiltonization relevant to hadronic charmless  $B$  decays can be written as [13]

$$\mathcal{H}_{\text{eff}} = \frac{G_f}{\sqrt{2}} \left[ \lambda_u (C_1 O_1^u + C_2 O_2^u) + \lambda_c (C_1 O_1^c + C_2 O_2^c) - \lambda_t \left( \sum_{i=3}^{10} C_i O_i + C_{7\gamma} O_{7\gamma} + C_{8g} O_{8g} \right) \right] + \text{H.c.}, \quad (1)$$

where  $\lambda_i = V_{ib} V_{iq}^*$  are CKM factors and  $C_i(\mu)$  are the effective Wilson coefficients which have been reliably evaluated

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to the next-to-leading logarithmic order. The effective operators  $O_i$  can be expressed as follows:

$$\begin{aligned}
O_1^u &= (\bar{u}b)_{V-A}(\bar{q}u)_{V-A}, \\
O_2^u &= (\bar{u}_\alpha b_\beta)_{V-A}(\bar{q}_\beta u_\alpha)_{V-A}, \\
O_1^c &= (\bar{c}b)_{V-A}(\bar{q}c)_{V-A}, \\
O_2^c &= (\bar{c}_\alpha b_\beta)_{V-A}(\bar{q}_\beta c_\alpha)_{V-A}, \\
O_{3(5)} &= (\bar{q}b)_{V-A} \sum_{q'} (\bar{q}'q')_{V-A(V+A)}, \\
O_{4(6)} &= (\bar{q}_\alpha b_\beta)_{V-A} \sum_{q'} (\bar{q}'_\beta q'_\alpha)_{V-A(V+A)}, \\
O_{7(9)} &= (\bar{q}b)_{V-A} \sum_{q'} e_{q'} (\bar{q}'q')_{V+A(V-A)}, \\
O_{8(10)} &= (\bar{q}_\alpha b_\beta)_{V-A} \sum_{q'} e_{q'} (\bar{q}'_\beta q'_\alpha)_{V+A(V-A)}, \\
O_{7\gamma} &= \frac{e}{8\pi^2} m_b \bar{q}_\alpha \sigma^{\mu\nu} (1 + \gamma_5) b_\alpha F_{\mu\nu}, \\
O_{8g} &= \frac{g}{8\pi^2} m_b \bar{q}_\alpha \sigma^{\mu\nu} (1 + \gamma_5) T_{\alpha\beta}^a b_\beta G_{\mu\nu}^a, \quad (2)
\end{aligned}$$

where  $q=d,s$  and  $q'$  denotes all the active quarks at the scale  $\mu = \mathcal{O}(m_b)$ , i.e.,  $q' = u, d, s, c, b$ .

### B. The factorizable amplitude for $\bar{B}_s \rightarrow VV$

To calculate the decay rate and branching ratios for  $\bar{B}_s \rightarrow VV$  decays, we need the hadronic matrix element for the local four fermion operators

$$\langle V_1(\lambda_1) V_2(\lambda_2) | (\bar{q}_2 q_3)_{V-A} (\bar{q}_1 b)_{V-A} | \bar{B}_s \rangle, \quad (3)$$

where  $\lambda_1, \lambda_2$  are the helicities of the final-state vector mesons  $V_1$  and  $V_2$  with four-momentum  $p_1$  and  $p_2$ , respectively. In the rest frame of the  $B_s$  system, since the  $B_s$  meson has spin zero, we have  $\lambda_1 = \lambda_2 = \lambda$ . Let  $X^{(B_s V_1, V_2)}$  denote the factorizable amplitude with the vector meson  $V_2$  being factored out, under the naive factorization (NF) approach, we can express  $X^{(B_s V_1, V_2)}$  as

$$X^{(B_s V_1, V_2)} = \langle V_2 | (\bar{q}_2 q_3)_{V-A} | 0 \rangle \langle V_1 | (\bar{q}_1 b)_{V-A} | \bar{B}_s \rangle. \quad (4)$$

In terms of the decay constant and form factors defined by [14–17]

$$\begin{aligned} \langle V(p, \varepsilon^*) | \bar{q} \gamma_\mu q' | 0 \rangle \\ = -if_V m_V \varepsilon_\mu^*, \end{aligned} \quad (5)$$

$$\begin{aligned} \langle V(p, \varepsilon^*) | \bar{q} \gamma_\mu (1 - \gamma_5) b | \bar{B}_s(p_B) \rangle \\ = -\varepsilon_\mu^* (m_B + m_V) A_1^{B_s V}(q^2) \\ + (p_B + p)_\mu (\varepsilon^* \cdot p_B) \frac{A_2^{B_s V}(q^2)}{m_B + m_V} \\ + q_\mu (\varepsilon^* \cdot p_B) \frac{2m_V}{q^2} [A_3^{B_s V}(q^2) - A_0^{B_s V}(q^2)] \\ - i \varepsilon_{\mu\nu\alpha\beta} \varepsilon^{*\nu} p_B^\alpha p^\beta \frac{2V^{B_s V}(q^2)}{m_B + m_V}, \end{aligned} \quad (6)$$

where  $q = p_B - p$  and the form factors obey the following exact relations:

$$A_3(0) = A_0(0),$$

$$A_3^{B_s V}(q^2) = \frac{m_B + m_V}{2m_V} A_1^{B_s V}(q^2) - \frac{m_B - m_V}{2m_V} A_2^{B_s V}(q^2). \quad (7)$$

With the above equations, the factorizable amplitude for  $\bar{B}_s \rightarrow V_1 V_2$  can be written as

$$\begin{aligned} X^{(\bar{B}_s V_1, V_2)} &= if_{V_2} m_{V_2} \left[ (\varepsilon_1^* \cdot \varepsilon_2^*) (m_{B_s} + m_{V_1}) A_1^{B_s V_1}(m_{V_2}^2) \right. \\ &\quad - (\varepsilon_1^* \cdot p_B) (\varepsilon_2^* \cdot p_B) \frac{2A_2^{B_s V_1}(m_{V_2}^2)}{m_{B_s} + m_{V_1}} \\ &\quad \left. + i \varepsilon_{\mu\nu\alpha\beta} \varepsilon_2^{*\mu} \varepsilon_1^{*\nu} p_B^\alpha p^\beta \frac{2V^{B_s V}(q^2)}{m_{B_s} + m_{V_1}} \right], \end{aligned} \quad (8)$$

where  $p_B(m_{B_s})$  is the four-momentum (mass) of the  $\bar{B}_s$  meson, and  $m_{V_1}(\varepsilon_1^*)$  and  $m_{V_2}(\varepsilon_2^*)$  are the masses (polarization vectors) of the two vector mesons  $V_1$  and  $V_2$ , respectively. Here and in throughout the remainder of the paper we use the sign convention  $\epsilon^{0123} = -1$ . Assuming the  $V_1(V_2)$  meson flying in the plus (minus)  $z$  direction carrying the momentum  $p_1(p_2)$ , we get

$$X^{(\bar{B}_s V_1, V_2)} = \begin{cases} \frac{-if_{V_2}}{2m_{V_1}} \left[ (m_{B_s}^2 - m_{V_1}^2 - m_{V_2}^2)(m_{B_s} + m_{V_1}) A_1^{B_s V_1}(m_{V_2}^2) - \frac{4m_{B_s}^2 p_c^2}{m_{B_s} + m_{V_1}} A_2^{B_s V_1}(m_{V_2}^2) \right] \equiv h_0 & \text{for } \lambda = 0, \\ -if_{V_2} m_{V_2} \left[ (m_{B_s} + m_{V_1}) A_1^{B_s V_1}(m_{V_2}^2) \pm \frac{2m_{B_s} p_c}{m_{B_s} + m_{V_1}} V^{B_s V_1}(m_{V_2}^2) \right] \equiv h_\pm & \text{for } \lambda = \pm, \end{cases} \quad (9)$$

where  $\lambda=0,\pm$  is the helicity of the vector meson and  $p_c = |\vec{p}_1| = |\vec{p}_2|$  is the momentum of either of the two outgoing vector mesons in the  $\bar{B}_s$  rest frame.

In general, the  $\bar{B}_s \rightarrow V_1 V_2$  amplitude can be decomposed into three independent helicity amplitudes  $H_0$ ,  $H_+$ , and  $H_-$ , corresponding to  $\lambda=0, +$ , and  $-$ , respectively. We use the notation

$$H_\lambda = \langle V_1(\lambda) V_2(\lambda) | \mathcal{H}_{\text{eff}} | \bar{B}_s \rangle \quad (10)$$

for the helicity matrix element and it can be expressed by three independent Lorentz scalars  $a$ ,  $b$ , and  $c$ . The relations between them can be written as [3,17]

$$H_\lambda = \varepsilon_{1\mu}^* \varepsilon_{2\nu}^* \left( a g^{\mu\nu} + \frac{b}{m_{V_1} m_{V_2}} p_B^\mu p_B^\nu + \frac{ic}{m_{V_1} m_{V_2}} \varepsilon^{\mu\nu\alpha\beta} p_{1\alpha} p_{2\beta} \right), \quad (11)$$

where the coefficient  $c$  corresponds to the  $p$ -wave amplitude, and  $a$  and  $b$  to the mixture of  $s$ - and  $d$ -wave amplitudes. The helicity amplitudes can be reconstructed as

$$H_0 = -ax - b(x^2 - 1), \quad (12)$$

$$H_\pm = -a \mp c \sqrt{x^2 - 1}, \quad (13)$$

where  $x = p_1 \cdot p_2 / m_{V_1} m_{V_2}$ . Given the helicity amplitudes, the decay rate and the branching ratio for  $\bar{B}_s \rightarrow V_1 V_2$  can be written as

$$\Gamma(\bar{B}_s \rightarrow V_1 V_2) = \frac{p_c}{8\pi m_{B_s}^2} (|H_0|^2 + |H_+|^2 + |H_-|^2),$$

$$Br(\bar{B}_s \rightarrow V_1 V_2) = \tau_{B_s} \frac{p_c}{8\pi m_{B_s}^2} (|H_0|^2 + |H_+|^2 + |H_-|^2), \quad (14)$$

where  $\tau_{B_s}$  is the lifetime of the  $B_s$  meson, and  $p_c$  is given by

$$p_c = \frac{1}{2m_{B_s}} \sqrt{[m_{B_s}^2 - (m_{V_1} + m_{V_2})^2][m_{B_s}^2 - (m_{V_1} - m_{V_2})^2]}. \quad (15)$$

### C. QCD factorization for $\bar{B}_s \rightarrow VV$

Under the naive factorization (NF) approach, the coefficients  $a_i$  are given by  $a_i = C_i + 1/N_C C_{i+1}$  for odd  $i$  and  $a_i = C_i + 1/N_C C_{i-1}$  for even  $i$ , which are obviously independent of the helicity  $\lambda$ . In the present paper, we will compute the nonfactorizable corrections to the effective parameters  $a_i^h$ , which, however, are not the same for different helicity amplitudes  $H_0$  and  $H_\pm$ .

The QCD-improved factorization (QCDF) approach advocated by Beneke *et al.* [3] allows us to compute the nonfactorizable corrections to the hadronic matrix elements

$\langle V_1 V_2 | O_i | \bar{B}_s \rangle$  in the heavy quark limit, since in the  $m_b \rightarrow \infty$  limit only hard interactions between the  $(\bar{B}_s V_1)$  system and  $V_2$  survive. In this method, the light-cone distribution amplitudes (LCDAs) play an essential role. Since we are only concerned with two light vector mesons in the final states, the LCDAs of the light vector meson of interest in momentum configuration are given by [15,17]

$$\mathcal{M}_{\delta\alpha}^V = \mathcal{M}_{\delta\alpha\parallel}^V + \mathcal{M}_{\delta\alpha\perp}^V \quad (16)$$

with (here we suppose the vector meson moving in the  $n_-$  direction)

$$\begin{aligned} \mathcal{M}_{\parallel}^V &= -\frac{if_V}{4} \frac{m_V(\varepsilon^* \cdot n_+)}{2} h_- \Phi_{\parallel}^V(u), \\ \mathcal{M}_{\perp}^V &= -\frac{if_V}{4} E \varepsilon_{\perp}^* h_- \Phi_{\perp}^V(u) \\ &\quad - \frac{if_V m_V}{4} \left[ \varepsilon_{\perp}^* g_{\perp}^{(v)V}(u) \right. \\ &\quad \left. + i \varepsilon_{\mu\nu\rho\sigma} \varepsilon_{\perp}^{*\nu} n_-^\rho n_+^\sigma \gamma^\mu \gamma_5 \frac{g_{\perp}^{(a)V}(u)}{8} \right], \end{aligned} \quad (17)$$

where  $n_{\pm} = (1, 0, 0, \pm 1)$  are the light-cone null vectors,  $u$  is the light-cone momentum fraction of the quark in the vector meson,  $f_V$  and  $f_{\perp}^V$  are vector and tensor decay constants, and  $E$  is the energy of the vector meson in the  $B_s$  rest system. In Eq. (17),  $\Phi_{\parallel}^V(u)$  and  $\Phi_{\perp}^V(u)$  are leading-twist distribution amplitudes (DAs), while  $g_{\perp}^{(v)V}(u)$  and  $g_{\perp}^{(a)V}(u) = dg_{\perp}^{(a)V}(u)/du$  are twist-3 ones. Since the twist-2 DA  $\Phi_{\perp}^V(u)$  contributions to the vertex corrections and hard spectator interactions vanish in the chiral limit, and furthermore, the contributions of the twist-3 DAs  $h_{\parallel}^{(s,t)}(u)$  are power suppressed compared to that of the leading twist ones for the helicity zero case, therefore we will work towards the leading-twist approximation for longitudinally polarized states and to the twist-3 level for transversely polarized ones. We note that the same observation has been made by Cheng and Yang [18] in studying  $B_{u,d} \rightarrow \phi k^*$ .

In the heavy quark limit, the light-cone projector for the  $B$  meson can be expressed as [5,19]

$$\mathcal{M}_{\alpha\beta}^B = -\frac{if_B m_B}{4} [(1 + \not{n}) \gamma_5 \{ \Phi_1^B(\xi) + \not{n} \Phi_2^B(\xi) \}]_{\beta\alpha}, \quad (18)$$

with the normalization condition

$$\int_0^1 d\xi \Phi_1^B(\xi) = 1, \quad \int_0^1 d\xi \Phi_2^B(\xi) = 0, \quad (19)$$

where  $\xi$  is the momentum fraction of the spectator quark in the  $B$  meson.

Equipped with these preliminaries, we can now calculate the nonfactorizable corrections to the effective parameters  $a_i^h$  systematically. After direct calculations, we get

$$\begin{aligned}
a_1^h &= C_1 + \frac{C_2}{N_C} + \frac{\alpha_s}{4\pi} \frac{C_F}{N_C} C_2 (f_I^h + f_{II}^h), \\
a_2^h &= C_2 + \frac{C_1}{N_C} + \frac{\alpha_s}{4\pi} \frac{C_F}{N_C} C_1 (f_I^h + f_{II}^h), \\
a_3^h &= C_3 + \frac{C_4}{N_C} + \frac{\alpha_s}{4\pi} \frac{C_F}{N_C} C_4 (f_I^h + f_{II}^h), \\
a_4^h &= C_4 + \frac{C_3}{N_C} + \frac{\alpha_s}{4\pi} \frac{C_F}{N_C} C_3 (f_I^h + f_{II}^h) \\
&\quad + \frac{\alpha_s}{4\pi} \frac{C_F}{N_C} \left[ \left( C_3 - \frac{1}{2} C_9 \right) \left[ G^h(s_q) + G^h(s_b) - \left( \frac{4}{3} \right) \right] \right. \\
&\quad \left. - C_1 \left[ \frac{\lambda_u}{\lambda_t} G^h(s_u) + \frac{\lambda_c}{\lambda_t} G^h(s_c) + \left( \frac{2}{3} \right) \right] \right. \\
&\quad \left. + (C_4 + C_6) \sum_{i=u}^b G^h(s_i) \right. \\
&\quad \left. + \frac{3}{2} (C_8 + C_{10}) \sum_{i=u}^b G^h(s_i) + C_{8g} G_g^h \right], \\
a_5^h &= C_5 + \frac{C_6}{N_C} - \frac{\alpha_s}{4\pi} \frac{C_F}{N_C} C_6 (f_I^h + f_{II}^h), \\
a_6^h &= C_6 + \frac{C_5}{N_C}, \\
a_7^h &= C_7 + \frac{C_8}{N_C} - \frac{\alpha_s}{4\pi} \frac{C_F}{N_C} C_8 (f_I^h + f_{II}^h), \\
a_8^h &= C_8 + \frac{C_7}{N_C}, \\
a_9^h &= C_9 + \frac{C_{10}}{N_C} + \frac{\alpha_s}{4\pi} \frac{C_F}{N_C} C_{10} (f_I^h + f_{II}^h), \\
a_{10}^h &= C_{10} + \frac{C_9}{N_C} + \frac{\alpha_s}{4\pi} \frac{C_F}{N_C} C_9 (f_I^h + f_{II}^h),
\end{aligned} \tag{20}$$

where  $C_F = (N_C^2 - 1)/2N_C$ , and  $N_C = 3$  is the number of colors,  $s_i = m_i^2/m_b^2$  and  $q = d, s$  (determined by the  $b \rightarrow d$  or  $b \rightarrow s$  transition process). The superscript  $h$  denotes the polarization of the vector meson (which is equivalent to  $\lambda$ , but for convenience we shall adopt  $h$  in the following) where  $h = 0$  denotes the helicity 0 state and  $h = \pm$  for the helicity  $\pm$  ones. In the expression  $a_4^h$ , the upper value in parentheses corresponds to the  $h = 0$  state, while the lower value to the  $h = \pm$  ones.

In Eq. (20),  $f_I^h$  denotes the contributions from the vertex corrections. In the naive dimensional regularization (NDR) scheme for  $\gamma_5$ , it is given by

$$\begin{aligned}
f_I^0 &= -12 \log \frac{\mu}{m_b} - 18 + \int_0^1 du \Phi_{\parallel}^2 V_2(u) \left( 3 \frac{1-2u}{1-u} - 3i\pi \right), \\
f_I^{\pm} &= -12 \log \frac{\mu}{m_b} - 16 + \int_0^1 du \left[ g_{\perp}^{(v)V_2}(u) \mp \frac{g_{\perp}^{\prime(a)V_2}(u)}{4} \zeta \right] \\
&\quad \times \left\{ \left( 3 \frac{1-2u}{1-u} \nu - 3i\pi \right) \right. \\
&\quad \left. + 2 \int_0^1 dx dy \left[ \frac{1-x-y}{xy} - \frac{u}{xu+y} \mp \frac{(1-x)u}{y(xu+y)} \right] \right\}, \tag{21}
\end{aligned}$$

where  $\zeta = +1$  or  $-1$ , corresponding to  $(V-A) \otimes (V-A)$  or  $(V-A) \otimes (V+A)$  current, respectively. It is obvious that  $f_I^0$  has the same expression as the hard scattering kernel  $F_{M_2}$  for the  $B \rightarrow \pi\pi$  mode [3,8] as it should be.

For hard spectator interactions, supposing  $V_1$  to be the recoiled meson and  $V_2$  the emitted meson, explicit calculations for  $f_{II}^h$  yields

$$\begin{aligned}
f_{II}^0 &= -\frac{4\pi^2}{N_C} \frac{2f_{B_s} f_{V_1} m_{V_1}}{m_{B_s}^3 [A_1^{B_s V_1}(0) - A_2^{B_s V_1}(0)]} \\
&\quad \times \int_0^1 d\xi \frac{\Phi_1^B(\xi)}{\xi} \int_0^1 dv \frac{\Phi_{\parallel}^{V_1}(v)}{\bar{v}} \int_0^1 du \frac{\Phi_{\parallel}^{V_2}(u)}{u}, \\
f_{II}^{\pm} &= \frac{4\pi^2}{N_C} \frac{f_{B_s} f_{\perp}^{V_1}}{m_{B_s}^2 [A_1^{B_s V_1}(0) \pm V^{B_s V_1}(0)]} 2(1 \pm 1) \\
&\quad \times \int_0^1 d\xi \frac{\Phi_1^B(\xi)}{\xi} \int_0^1 dv \frac{\Phi_{\perp}^{V_1}(v)}{\bar{v}} \\
&\quad \times \int_0^1 du \left( g_{\perp}^{(v)V_2}(u) \mp \frac{g_{\perp}^{\prime(a)V_2}(u)}{4} \zeta \right) \\
&\quad - \frac{4\pi^2}{N_C} \frac{2f_{B_s} f_{V_1} m_{V_1}}{m_{B_s}^3 [A_1^{B_s V_1}(0) \pm V^{B_s V_1}(0)]} \int_0^1 d\xi \frac{\Phi_1^B(\xi)}{\xi} \\
&\quad \times \int_0^1 dv du \left( g_{\perp}^{(v)V_1}(u) \mp \frac{g_{\perp}^{\prime(a)V_1}(u)}{4} \right) \\
&\quad \times \left( g_{\perp}^{(v)V_2}(u) \mp \frac{g_{\perp}^{\prime(a)V_2}(u)}{4} \zeta \right) \frac{u + \bar{v}}{u\bar{v}^2}, \tag{22}
\end{aligned}$$

with  $\bar{v} = 1 - v$ . In Eq. (22), when we adopt the asymptotical form for the vector meson LCDAs, there will be a logarithmic infrared divergence with regard to the  $v$  integral in  $f_{II}^{\pm}$ , which implies that the spectator interaction is dominated by soft gluon exchanges in the final states. In analogy with the treatment in works [6,10], we parametrize it as

$$X_h = \int_0^1 = \log \frac{m_b}{\Lambda_h} (1 + \rho_H e^{i\phi_H}), \tag{23}$$

with  $(\rho_H, \phi_H)$  related to the contributions from hard spectator scattering. Since the parameters  $(\rho_H, \phi_H)$  are unknown, how to treat them is a major theoretical uncertainty in the QCD factorization approach. In the later numerical analysis, we shall take  $\Lambda_h = 0.5$  GeV,  $(\rho_h, \phi_h) = (0, 0)$  [11] as our default values.

In calculating the contributions of the QCD penguin-type diagrams, we should pay attention to the fact that there are two distinctly different contractions argued in [5]. With this in mind, the nonfactorizable corrections induced by local four-quark operators  $O_i$  can be described by the function  $G^h(s)$  which is given by

$$\begin{aligned} G^0(s) &= -\frac{2}{3} + \frac{4}{3} \log \frac{\mu}{m_b} - 4 \int_0^1 du \Phi_{\parallel}^{V_2}(u) g(u, s), \\ G^{\pm}(s) &= -\frac{2}{3} + \frac{2}{3} \log \frac{\mu}{m_b} \\ &\quad - 2 \int_0^1 du \left( g_{\perp}^{(v)V_2}(u) \mp \frac{g_{\perp}^{\prime(a)V_2}(u)}{4} \right) g(u, s), \end{aligned} \quad (24)$$

with the function

$$g(u, s) = \int_0^1 dx x \bar{x} \log[s - x\bar{x}(1-u)]. \quad (25)$$

In Eq. (20), we also take into account the contributions of the dipole operator  $O_{8g}$  which will give a tree-level contribution described by the function  $G_g^h$  defined as

$$\begin{aligned} G_g^0 &= \int_0^1 du \frac{2\Phi_{\parallel}^{V_2}(u)}{1-u}, \\ G_g^+ &= \int_0^1 du \left( g_{\perp}^{(v)V_2}(u) - \frac{g_{\perp}^{\prime(a)V_2}(u)}{4} \right), \\ G_g^- &= \int_0^1 du \left( g_{\perp}^{(v)V_2}(u) - \frac{g_{\perp}^{\prime(a)V_2}(u)}{4} \right) \frac{1}{1-u}. \end{aligned} \quad (26)$$

Due to  $\langle V | \bar{q}_1 q_2 | 0 \rangle = 0$ ,  $\bar{B}_s \rightarrow V_1 V_2$  decays do not receive nonfactorizable contributions from  $a_6^h$  and  $a_8^h$  penguin terms as shown in Eq. (20).

### III. NUMERICAL RESULTS AND DISCUSSIONS

To proceed, we use the next-to-leading order Wilson coefficients in the NDR scheme for  $\gamma_5$  [13]

$$\begin{aligned} C_1 &= 1.078, & C_2 &= -0.176, & C_3 &= 0.014, \\ C_4 &= -0.034, & C_5 &= 0.008, & C_6 &= -0.039, \\ C_7/\alpha &= -0.011, & C_8/\alpha &= 0.055, & C_9/\alpha &= -1.341, \\ C_{10}/\alpha &= 0.264, & C_{8g} &= -0.146, \end{aligned} \quad (27)$$

at  $\mu = m_b = 4.66$  GeV, with  $\alpha$  being the electromagnetic fine-structure coupling constant. For quark masses, which appear in the penguin loop corrections with regard to the functions  $G^h(s)$ , we take

$$m_u = m_d = m_s = 0, \quad m_c = 1.47 \text{ GeV}, \quad m_b = 4.66 \text{ GeV}. \quad (28)$$

As for the CKM matrix elements, we adopt the Wolfenstein parametrization up to  $\mathcal{O}(\lambda^3)$ :

$$\begin{aligned} &\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \\ &= \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix}. \end{aligned} \quad (29)$$

For the Wolfenstein parameters appearing in the above expression, we shall use the values given by [20]

$$\lambda = 0.2236, \quad A = 0.824, \quad \bar{\rho} = 0.22, \quad \bar{\eta} = 0.35, \quad (30)$$

where  $\bar{\rho} = \rho(1 - \lambda^2/2)$  and  $\bar{\eta} = \eta(1 - \lambda^2/2)$ . For computing the branching ratio, the lifetime of the  $B_s$  meson is  $\tau_{B_s} = 1.461$  ps [20].

For the LCDAs of the vector meson, we use the asymptotic form [17]

$$\begin{aligned} \Phi_{\parallel}^V(x) &= \Phi_{\perp}^V(x) = g_{\perp}^{(a)V} = 6x(1-x), \\ g_{\perp}^{(v)V}(x) &= \frac{3}{4} [1 + (2x-1)^2]. \end{aligned} \quad (31)$$

As for the two  $B_s$  meson wave functions given by Eq. (18), we find that only  $\Phi_1^B(\xi)$  has contributions to the nonfactorizable corrections. We adopt the moments of the  $\Phi_1^B(\xi)$  defined by [3,5] in our numerical evaluation

$$\int_0^1 d\xi \frac{\Phi_1^B(\xi)}{\xi} = \frac{m_{B_s}}{\Lambda_B}, \quad (32)$$

with  $\Lambda_B = 0.35$  GeV. The quantity  $\Lambda_B$  parametrizes our ignorance about the  $B_s$  meson distribution amplitudes and thus brings large theoretical uncertainty.

The decay constants and form factors are nonperturbative parameters which are taken as input parameters. In principle, they are available from the experimental data and/or estimated with well-founded theories, such as lattice calculations, QCD sum rules, etc. For the decay constants, we take their values in our calculations as [11,21,14]

$$\begin{aligned} f_{B_s} &= 236 \text{ MeV}, & f_{K^*} &= 214 \text{ MeV}, & f_{\rho} &= 210 \text{ MeV}, \\ f_{\omega} &= 195 \text{ MeV}, & f_{\phi} &= 233 \text{ MeV}. \end{aligned} \quad (33)$$



TABLE I. The effective parameters  $a_i^h$  in the NF and QCDF approaches for  $\bar{B}_s \rightarrow K^{+*} \rho^-$ .

| $a_i^h$    | NF     | QCDF              |
|------------|--------|-------------------|
| $a_1^0$    | 1.019  | $1.778 + 0.113i$  |
| $a_4^0$    | -0.029 | $-0.054 - 0.014i$ |
| $a_{10}^0$ | -0.001 | $1.411 - 0.429i$  |
| $a_1^+$    | 1.019  | $0.924 + 0.113i$  |
| $a_4^+$    | -0.029 | $-0.068 - 0.013i$ |
| $a_{10}^+$ | -0.001 | $-0.375 + 0.139i$ |
| $a_1^-$    | 1.019  | $1.281 + 0.113i$  |
| $a_4^-$    | -0.029 | $-0.068 + 0.020i$ |
| $a_{10}^-$ | -0.001 | $-0.034 + 0.573i$ |

For the form factors involving the  $B_s \rightarrow K^*$  and  $B_s \rightarrow \phi$  transition, we adopt the results given by [14] which are analyzed using the light-cone sum rule (LCSR) method with the parametrization

$$f(q^2) = \frac{f(0)}{1 - a_F(q^2/m_B^2) + b_F(q^2/m_B^2)^2} \quad (34)$$

for the form-factor  $q^2$  dependence. At the maximum recoil, the form factors are listed as [14]

$$\begin{aligned} A_1^{B_s \phi}(0) &= 0.296, & a_F &= 0.87, & b_F &= -0.061, \\ A_2^{B_s \phi}(0) &= 0.255, & a_F &= 1.55, & b_F &= 0.513, \\ V^{B_s \phi}(0) &= 0.433, & a_F &= 1.75, & b_F &= 0.736, \\ A_1^{B_s K^*}(0) &= 0.190, & a_F &= 1.02, & b_F &= -0.037, \\ A_2^{B_s K^*}(0) &= 0.164, & a_F &= 1.77, & b_F &= 0.729, \\ V^{B_s K^*}(0) &= 0.262, & a_F &= 1.89, & b_F &= 0.846. \end{aligned} \quad (35)$$

It is obvious that the  $q^2$  dependence for the form factors  $A_2$  and  $V$  is dominated by the dipole terms, while the  $A_1$  is dominated by the monopole term in the region where  $q^2$  is not too large.

To illustrate the nonuniversality of the nonfactorizable effects on different helicity amplitudes, we list a few numerical results of the parameters  $a_i^h$  for a specific mode  $\bar{B}_s \rightarrow K^{+*} \rho^-$  in Table I. In order to compare with the parameters  $a_i$  in the NF approach, we also present the results of  $a_i$  calculated in the QCDF approach.

From Table I, we can see that nonfactorizable corrections to the helicity amplitudes are not universal. The effective parameters  $a_i^h$  for helicity  $h=0, +, -$  states receive different nonfactorizable contributions and hence they are helicity dependent, quite contrary to the naive factorization (NF) approach where the parameters  $a_i$  are universal and polarization independent.

TABLE II. Branching ratios and the transverse to total decay rate  $\Gamma_T/\Gamma$  for charmless  $\bar{B}_s \rightarrow VV$  decays in the QCD factorization (QCDF) approach and in the NF approach.

| Channel                                     | $\Gamma_T/\Gamma$ |       | BR                    |                       |
|---|-------------------|-------|-----------------------|-----------------------|
|   | QCDF              | NF    | QCDF                  | NF                    |
| <i>b</i> → <i>d</i> transition              |                   |       |                       |                       |
| $\bar{B}_s \rightarrow K^{+*} \rho^-$       | 0.056             | 0.066 | $2.17 \times 10^{-5}$ | $1.88 \times 10^{-5}$ |
| $\bar{B}_s \rightarrow K^{0*} \rho^0$       | 0.045             | 0.062 | $1.95 \times 10^{-6}$ | $6.29 \times 10^{-7}$ |
| $\bar{B}_s \rightarrow K^{0*} \omega$       | 0.049             | 0.064 | $1.41 \times 10^{-6}$ | $7.50 \times 10^{-7}$ |
| <i>b</i> → <i>s</i> transition              |                   |       |                       |                       |
| $\bar{B}_s \rightarrow K^{+*} K^{*-}$       | 0.084             | 0.103 | $2.10 \times 10^{-6}$ | $1.58 \times 10^{-6}$ |
| $\bar{B}_s \rightarrow \omega \phi$         | 0.072             | 0.072 | $1.30 \times 10^{-6}$ | $2.64 \times 10^{-7}$ |
| $\bar{B}_s \rightarrow \rho^0 \phi$         | 0.038             | 0.070 | $1.67 \times 10^{-6}$ | $7.14 \times 10^{-7}$ |
| pure penguin processes                      |                   |       |                       |                       |
| $\bar{B}_s \rightarrow K^{0*} \bar{K}^{0*}$ | 0.044             | 0.082 | $3.72 \times 10^{-6}$ | $1.94 \times 10^{-6}$ |
| $\bar{B}_s \rightarrow K^{0*} \phi$         | 0.113             | 0.092 | $1.98 \times 10^{-7}$ | $1.40 \times 10^{-7}$ |
| $\bar{B}_s \rightarrow \phi \phi$           | 0.084             | 0.117 | $3.68 \times 10^{-5}$ | $1.79 \times 10^{-5}$ |

The branching ratios for several channels of  $\bar{B}_s \rightarrow VV$  decays in the LCSR analysis for form factors are collected in Table II. In order to compare the size of different helicity amplitudes, we define two quantities:

$$\frac{\Gamma_T}{\Gamma} = \frac{|H_+|^2 + |H_-|^2}{|H_0|^2 + |H_+|^2 + |H_-|^2}, \quad (36)$$

$$\frac{\Gamma_L}{\Gamma} = \frac{|H_+|^2 + |H_-|^2}{|H_0|^2 + |H_+|^2 + |H_-|^2}. \quad (37)$$

The ratios of  $\Gamma_T/\Gamma$  and  $\Gamma_L/\Gamma$  measure the relative amount of a transversely and longitudinally polarized vector meson. In Table II we also give the values of  $\Gamma_T/\Gamma$  for each channel both in the QCD factorization (QCDF) approach and the naive factorization (NF) approach.

From Table II, we can find that some channels have large branching ratios of order  $10^{-5}$ , which are measurable at near future experiments at CERN LHCb. Owing to the absence of  $(S-P)(S+P)$  penguin operator contributions to  $W$ -emission amplitudes, tree-dominated  $\bar{B}_s \rightarrow V_1 V_2$  decays tend to have larger branching ratios than the penguin-dominated ones. Moreover, we find that the transverse to total decay rate  $\Gamma_T/\Gamma$  is very small for most decay modes, so in the heavy quark limit, both light vector mesons in charmless  $\bar{B}_s \rightarrow VV$  decays tend to have zero helicity.

#### IV. SUMMARY

In this paper, we calculated the branching ratios for two-body charmless hadronic  $\bar{B}_s \rightarrow VV$  decays within the framework of QCD factorization. Contrary to phenomenological generalized factorization [9] and the NF approach, the nonfactorizable corrections to each helicity amplitude are not the

same. The effective parameters  $a_i^h$  vary for different helicity amplitude and hence are helicity dependent. Since the leading-twist DAs contributions to the transversely polarized amplitudes vanish in the chiral limit, in order to have renormalization scale and scheme independent predictions it is necessary to take into account the contributions of the twist-3 DAs of the vector meson. Contrary to the  $PP$  and  $PV$  modes, the annihilation amplitudes in the  $VV$  case do not gain the chiral enhancement of order  $m_b^2/(m_q m_b)$ . So we do not include the contributions of the annihilation diagrams which are truly power suppressed in the heavy quark limit. It should be stressed that we have not taken into account the higher-twist DAs contribution for the longitudinally polarized vector meson. Through direct calculation, the transverse to total decay rate  $\Gamma_T/\Gamma$  is found to be very small, and both light vector mesons tend to have zero helicity. Branching ratios of  $\bar{B}_s \rightarrow VV$  decays are calculated with the LCSR analysis for the form factors and the branching ratios of some channels are found as large as  $10^{-5}$ , which might be accessible at future experiments at CERN LHCb.

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### APPENDIX

The  $\bar{B}_s \rightarrow VV$  decay amplitudes are collected here.

#### 1. $b \rightarrow d$ processes

$$\begin{aligned}
 H^h(\bar{B}_s \rightarrow K^{+*} \rho^-) &= \frac{G_F}{\sqrt{2}} \{ \lambda_u a_1^h - \lambda_t (a_4^h + a_{10}^h) \} X^{(B_s K^{+*}, \rho^-)}, \\
 H^h(\bar{B}_s \rightarrow K^{0*} \rho^0) &= \frac{G_F}{\sqrt{2}} \left\{ \lambda_u a_2^h - \lambda_t \left( -a_4^h + \frac{3}{2} a_7^h + \frac{3}{2} a_9^h \right. \right. \\
 &\quad \left. \left. + \frac{1}{2} a_{10}^h \right) \right\} X^{(B_s K^{0*}, \rho^0)}, \quad (\text{A1}) \\
 H^h(\bar{B}_s \rightarrow K^{0*} \omega) &= \frac{G_F}{\sqrt{2}} \left\{ \lambda_u a_2^h - \lambda_t \left( 2a_3^h + a_4^h + 2a_5^h + \frac{1}{2} a_7^h \right. \right. \\
 &\quad \left. \left. + \frac{1}{2} a_9^h - \frac{1}{2} a_{10}^h \right) \right\} X^{(B_s K^{0*}, \omega)},
 \end{aligned}$$

where  $\lambda_u = V_{ub} V_{ud}^*$  and  $\lambda_t = V_{tb} V_{td}^*$ .

#### 2. $b \rightarrow s$ processes

$$\begin{aligned}
 H^h(\bar{B}_s \rightarrow K^{+*} K^{*-}) &= \frac{G_F}{\sqrt{2}} \{ \lambda_u a_1^h - \lambda_t (a_4^h + a_{10}^h) \} \\
 &\quad \times X^{(B_s K^{+*}, K^{*-})}, \\
 H^h(\bar{B}_s \rightarrow \rho^0 \phi) &= \frac{G_F}{\sqrt{2}} \left\{ \lambda_u a_2^h - \lambda_t \right. \\
 &\quad \left. \times \left[ \frac{3}{2} (a_7^h + a_9^h) \right] \right\} X^{(B_s \phi, \rho^0)}, \quad (\text{A2}) \\
 H^h(\bar{B}_s \rightarrow \omega \phi) &= \frac{G_F}{\sqrt{2}} \left\{ \lambda_u a_2^h - \lambda_t \left[ 2(a_3^h + a_5^h) \right. \right. \\
 &\quad \left. \left. + \frac{1}{2} (a_7^h + a_9^h) \right] \right\} X^{(B_s \phi, \omega)},
 \end{aligned}$$

where  $\lambda_u = V_{ub} V_{us}^*$  and  $\lambda_t = V_{tb} V_{ts}^*$ .

#### 3. Pure penguin processes

$$\begin{aligned}
 H^h(\bar{B}_s \rightarrow K^{0*} \bar{K}^{0*}) &= -\frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \left( a_4^h - \frac{1}{2} a_{10}^h \right) X^{(B_s K^{0*}, \bar{K}^{0*})}, \\
 H^h(\bar{B}_s \rightarrow K^{0*} \phi) &= -\frac{G_F}{\sqrt{2}} V_{tb} V_{td}^* \left\{ \left[ a_3^h + a_5^h - \frac{1}{2} (a_7^h + a_9^h) \right] \right. \\
 &\quad \left. \times X^{(B_s K^{0*}, \phi)} + \left( a_4^h - \frac{1}{2} a_{10}^h \right) X^{(B_s \phi, K^{0*})} \right\}, \quad (\text{A3}) \\
 H^h(\bar{B}_s \rightarrow \phi \phi) &= -\frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \left[ a_3^h + a_4^h + a_5^h \right. \\
 &\quad \left. - \frac{1}{2} (a_7^h + a_9^h + a_{10}^h) \right] X^{(B_s \phi, \phi)}.
 \end{aligned}$$

In the above expressions, the factorizable amplitude  $X^{(B_s V_1, V_2)}$  is defined as in Eq. (8).

- [1] BABAR Collaboration, <http://www-public.slac.stanford.edu/babar/BaBarPublications>.  
[2] Belle Collaboration, <http://belle.kek.jp/>.  
[3] M. Beneke, G. Buchalla, M. Neubert, and C.T. Sachrajda, Phys. Rev. Lett. **83**, 1914 (1999); Nucl. Phys. **B591**, 313 (2000).

- [4] Y.Y. Keum, H.N. Li, and A.I. Sanda, Phys. Lett. B **504**, 6 (2001); Phys. Rev. D **63**, 054008 (2001).  
[5] M. Beneke, G. Buchalla, M. Neubert, and C.T. Sachrajda, Nucl. Phys. **B606**, 245 (2001); M. Beneke and M. Neubert, hep-ph/0308039.  
[6] D.S. Du, D.S. Yang, and G.H. Zhu, Phys. Lett. B **488**, 46

- (2000); D.S. Du, H.J. Gong, J.F. Sun, D.S. Yang, and G.H. Zhu, Phys. Rev. D **65**, 094025 (2002); D.S. Du, J.F. Jun, D.S. Yang, and G.H. Zhu, *ibid.* **67**, 014023 (2003).
- [7] H.Y. Cheng and K.C. Yang, Phys. Rev. D **64**, 074004 (2001).
- [8] T. Muta, A. Sugamoto, M.Z. Yang, and Y.D. Yang, Phys. Rev. D **62**, 094020 (2000); M.Z. Yang and Y.D. Yang, Nucl. Phys. **B609**, 245 (2001); Phys. Rev. D **62**, 114019 (2000).
- [9] Y.Y. Keum and H.N. Li, Phys. Rev. D **63**, 074006 (2001).
- [10] D. Zhang, Z. Xiao, and C.S. Li, Phys. Rev. D **64**, 014014 (2001).
- [11] J.F. Sun, G.H. Zhu, and D.S. Du, Phys. Rev. D **68**, 054003 (2003).
- [12] B. Tseng, Phys. Lett. B **446**, 125 (1999).
- [13] G. Buchalla, A.J. Buras, and M.E. Lautebacher, Rev. Mod. Phys. **68**, 1125 (1996).
- [14] P. Ball and V.M. Braun, Phys. Rev. D **58**, 094016 (1998); P. Ball, J. High Energy Phys. **09**, 005 (1998).
- [15] M. Beneke and T. Feldmann, Nucl. Phys. **B592**, 3 (2001).
- [16] J.M. Gerard and W.S. Hou, Phys. Rev. D **43**, 2909 (1991).
- [17] P. Ball and V.M. Braun, Nucl. Phys. **B543**, 201 (1999).
- [18] H.Y. Cheng and K.C. Yang, Phys. Lett. B **511**, 40 (2000).
- [19] A.G. Grozin and M. Neubert, Phys. Rev. D **55**, 272 (1997).
- [20] K. Hagiwara *et al.*, Phys. Rev. D **66**, 010001 (2002).
- [21] A. Ali, G. Kramer, and C.D. Lü, Phys. Rev. D **58**, 094009 (1998); **59**, 014005 (1999).