Charmless hadronic two-body decays of B_s mesons

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Two-body charmless nonleptonic decays of the B_s meson are studied within the framework of generalized factorization in which factorization is applied to the tree level matrix elements while the effective Wilson coefficients are μ and renormalization scheme independent, and nonfactorizable effects are parametrized in terms of $N_c^{\rm eff}(LL)$ and $N_c^{\rm eff}(LR)$, the effective numbers of colors arising from $(V-A)(V-A)$ and $(V-A)(V+A)$ four-quark operators, respectively. Branching ratios of $B_s \to PP, PV, VV$ decays (P: pseudoscalar meson, V: vector meson) are calculated as a function of $N_c^{\rm eff}(LR)$ with two different considerations for $N_c^{\rm eff}(LL)$: (a) $N_c^{\rm eff}(LL)$ being fixed at the value of 2 and (b) $N_c^{\rm eff}(LL)=N_c^{\rm eff}(LR)$. Tree and penguin transitions are classified into six different classes. We find the following. (i) The electroweak penguin contributions account for about 85% [for $N_c^{\rm eff}(LL)=2$] of the decay rates of $B_s \to \eta\pi,\ \eta'\pi,\ \eta\rho,\ \eta'\rho,\ \phi\pi,\ \phi\rho$, which receive contributions only from tree and electroweak penguin diagrams; a measurement of them will provide a clean determination of the electroweak penguin coefficient a_9. (ii) Electroweak penguin corrections to $B_s \to \omega\eta^{(\prime)}, \phi\eta, \omega\phi, K^{(*)}\phi, \phi\phi$ are in general as significant as QCD penguin effects and even play a dominant role; their decay rates depend strongly on $N_c^{\rm eff}(LR)$. (iii) The branching ratio of $B_s \to \eta\eta'$, the analogue of $B_d \to \eta' K$, is of order 2×10^{-5}, which is only slightly larger than that of $\eta'\eta', K^{*+}\rho^-, K^+K^-, K^0\overline{K}^0$ decay modes. (iv) The contribution from the η' charm content is important for $B_s \to \eta'\eta'$, but less significant for $B_s \to \eta\eta'$. (v) The decay rates for the final states $K^{+(*)}K^{-(*)}$ follow the pattern $\Gamma(\overline{B}_s \to K^+K^-) > \Gamma(\overline{B}_s \to K^+K^{*-}) \gtrsim \Gamma(\overline{B}_s \to K^{*+}K^{*-}) > \Gamma(\overline{B}_s \to K^{+*}K^-)$ and likewise for $K^{0(*)}\overline{K}^{0(*)}$, as a consequence of various interference effects between the penguin amplitudes governed by the effective QCD penguin coefficients a_4 and a_6. [S0556-2821(99)01405-8]

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I. INTRODUCTION

Recently there has been remarkable progress in the study of exclusive charmless B decays, both experimentally and theoretically. On the experimental side, CLEO has discovered many new two-body decay modes [1],

$$B \to \eta' K^+,\ \ \eta' K_S^0,\ \ \pi^\pm K_S^0,\ \ \pi^\pm K^\mp,\ \ \pi^0 K^\pm,\ \ \omega K^\pm, \tag{1}$$

and possible evidence for $B \to \phi K^*$. Moreover, CLEO has improved the upper limits for many other channels. Therefore, it is a field whose time has finally arrived. On the theoretical aspect, many important issues have been studied in past years, such as the effective Wilson coefficients that are renormalization scale and scheme independent, nonfactorizable effects in hadronic matrix elements, the QCD anomaly effect in the matrix element of pseudoscalar densities, running light quark masses at the scale m_b, and the q^2 dependence of form factors.

In the present paper, we plan to extend previous studies of charmless hadronic decays of B_u, B_d mesons to the B_s mesons. In principle, the physics for the B_s two-body hadronic decays is very similar to that for the B_d meson except that the spectator d quark is replaced by the s quark. Experimentally, it is known that $B^\pm \to \eta' K^\pm$ and $B_d \to \eta' K$ have abnormally large branching ratios, several times larger than previous predictions. It would be very interesting to see if the analogue of $B_d \to \eta' K$, namely, $B_s \to \eta\eta'$ or $B_s \to \eta'\eta'$ still has the largest branching ratio in two-body B_s charmless decays. Another point of interest is concerned with the electroweak penguin corrections. It is naively believed that in charmless B decays, the contributions from the electroweak penguin diagrams are negligible compared to the QCD penguin corrections because of smallness of electroweak penguin Wilson coefficients. As pointed out in Ref. [2], some B_s decay modes receive contributions only from the tree and electroweak penguin diagrams and moreover they are dominated by the latter. Therefore, electroweak penguins do play a dominant role in some of B_s decays. There also exist several penguin-dominated B_s decay modes in which electroweak penguin corrections to the decay rate are comparable to that of QCD penguin contributions. In this paper, we will study this in details.

Experimentally, only upper limits on the branching ratios have been established for a few B_s rare decay modes (see Ref. [3] or Table 7 of Ref. [1]) and most of them are far beyond the theoretical expectations. Nevertheless, it is conceivable that many of the B_s charmless decays can be seen at the future hadron colliders with large b production. Theoretically, early systematical studies can be found in Refs. [4,5]. More recently, one of us (B.T.) [6] has analyzed the exclusive charmless B_s decays involving the η or η' within the framework of generalized factorization.

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This paper is organized as follows. A calculational framework is set up in Sec. II in which we discuss the scale and scheme independent Wilson coefficient functions, parametrization of nonfactorizable effects, classification of factorizable amplitudes, etc. The numerical results and discussions are presented in Sec. III. Conclusions are summarized in Sec. IV. The factorizable amplitudes for all the charmless two-body B_s decays are given in Appendixes.

II. CALCULATIONAL FRAMEWORK

A. Effective Hamiltonian

The relevant effective $\Delta B=1$ weak Hamiltonian for hadronic charmless B decays is

$$\mathcal{H}_{\rm eff}(\Delta B=1)=\frac{G_F}{\sqrt{2}}\Bigg[V_{ub}V_{uq}^*(c_1O_1^u+c_2O_2^u)+V_{cb}V_{cq}^*(c_1O_1^c+c_2O_2^c)-V_{tb}V_{tq}^*\sum_{i=3}^{10}c_iO_i\Bigg]+\text{H.c.}, \tag{2}$$

where $q=d,s$, and

$$O_1^u=(\bar{u}b)_{V-A}(\bar{q}u)_{V-A}, \quad O_2^u=(\bar{q}b)_{V-A}(\bar{u}u)_{V-A},$$
$$O_{3(5)}=(\bar{q}b)_{V-A}\sum_{q'}(\bar{q}'q')_{V-A(V+A)},$$
$$O_{4(6)}=(\bar{q}_\alpha b_\beta)_{V-A}\sum_{q'}(\bar{q}'_\beta q'_\alpha)_{V-A(V+A)}, \tag{3}$$
$$O_{7(9)}=\frac{3}{2}(\bar{q}b)_{V-A}\sum_{q'}e_{q'}(\bar{q}'q')_{V+A(V-A)},$$
$$O_{8(10)}=\frac{3}{2}(\bar{q}_\alpha b_\beta)_{V-A}\sum_{q'}e_{q'}(\bar{q}'_\beta q'_\alpha)_{V+A(V-A)},$$

with O_3–O_6 being the QCD penguin operators, O_7–O_{10} the electroweak penguin operators, and $(\bar{q}_1q_2)_{V\pm A}\equiv\bar{q}_1\gamma_\mu(1\pm\gamma_5)q_2$. In order to ensure the renormalization-scale and -scheme independence for the physical amplitude, the matrix element of four-quark operators has to be evaluated in the same renormalization scheme as that for Wilson coefficients $c_i(\mu)$ and renormalized at the same scale μ. Generically, the hadronic matrix element is related to the tree level one via

$$\langle O(\mu)\rangle=g(\mu)\langle O\rangle_{\rm tree}, \tag{4}$$

with $g(\mu)$ being the perturbative corrections to the four-quark operators renormalized at the scale μ. We employ the relation (4) to write $\langle\mathcal{H}_{\rm eff}\rangle=c^{\rm eff}\langle O\rangle_{\rm tree}$. Schematically, the effective Wilson coefficients are given by $c^{\rm eff}=c(\mu)g(\mu)$. Formally, one can show that $c_i^{\rm eff}$ are μ and renormalization scheme independent. It is at this stage that the factorization approximation is applied to the hadronic matrix elements of the operator O at the tree level. The physical amplitude obtained in this manner is guaranteed to be renormalization scheme and scale independent.[1]

Perturbative QCD and electroweak corrections to $g(\mu)$ from vertex diagrams and penguin diagrams have been calculated in Refs. [8–11]. The penguin-type corrections depend on k^2, the gluon's momentum squared, and so do the effective Wilson coefficient functions. To the next-to-leading order, we obtain [12]

$$c_1^{\rm eff}=1.149, \quad c_2^{\rm eff}=-0.325,$$
$$c_3^{\rm eff}=0.0211+i0.0045, \quad c_4^{\rm eff}=-0.0450-i0.0136,$$
$$c_5^{\rm eff}=0.0134+i0.0045, \quad c_6^{\rm eff}=-0.0560-i0.0136,$$
$$c_7^{\rm eff}=-(0.0276+i0.0369)\alpha, \quad c_8^{\rm eff}=0.054\alpha,$$
$$c_9^{\rm eff}=-(1.318+i0.0369)\alpha, \quad c_{10}^{\rm eff}=0.263\alpha, \tag{5}$$

at $k^2=m_b^2/2$. It is interesting to note that $c_{1,2}^{\rm eff}$ are very close to the leading order Wilson coefficients: $c_1^{\rm LO}=1.144$ and $c_2^{\rm LO}=-0.308$ at $\mu=m_b(m_b)$ [13] and that $\text{Re}(c_{3-6}^{\rm eff})\approx\frac{3}{2}c_{3-6}^{\rm LO}(\mu)$. Therefore, the decay rates of charmless B decay modes dominated by QCD penguin diagrams will be too small by a factor of $\sim(1.5)^2=2.3$ if only leading-order penguin coefficients are employed for the calculation.

B. Parametrization of nonfactorizable effects

Because there is only one single form factor (or Lorentz scalar) involved in the class-I or class-II decay amplitude of $B\to PP,\ PV$ decays (see Sec. II C for the classification of factorizable amplitudes), the effects of nonfactorization can be lumped into the effective parameters a_1 and a_2 [14]:

$$a_1^{\rm eff}=c_1^{\rm eff}+c_2^{\rm eff}\left(\frac{1}{N_c}+\chi_1\right), \quad a_2^{\rm eff}=c_2^{\rm eff}+c_1^{\rm eff}\left(\frac{1}{N_c}+\chi_2\right), \tag{6}$$

where χ_i are nonfactorizable terms and receive main contributions from color-octet current operators. Since $|c_1^{\rm eff}/c_2^{\rm eff}|\gg1$, it is evident from Eq. (6) that even a small amount of nonfactorizable contributions will have a significant effect on the color-suppressed class-II amplitude. If $\chi_{1,2}$ are universal (i.e., process independent) in charm or bottom decays, then we have a generalized factorization scheme in which the decay amplitude is expressed in terms of factorizable contributions multiplied by the universal effective parameters $a_{1,2}^{\rm eff}$. For $B\to VV$ decays, this new factorization implies that nonfactorizable terms contribute in equal weight to all partial wave amplitudes so that $a_{1,2}^{\rm eff}$ *can* be defined. It should be stressed that, contrary to the naive one, the improved factor-

[1]This formulation is different from the one advocated in Ref. [7] in which the μ dependence of the Wilson coefficients $c_i(\mu)$ are assumed to be canceled out by that of the nonfactorization parameters $\varepsilon_8(\mu)$ and $\varepsilon_1(\mu)$ so that the effective parameters $a_i^{\rm eff}$ are μ independent.

ization does incorporate nonfactorizable effects in a process independent form. For example, $\chi_1=\chi_2=-\frac{1}{3}$ in the large-N_c approximation of factorization. Phenomenological analyses of the two-body decay data of D and B mesons indicate that while the generalized factorization hypothesis in general works reasonably well, the effective parameters $a_{1,2}^{\rm eff}$ do show some variation from channel to channel, especially for the weak decays of charmed mesons [14–16]. An eminent feature emerged from the data analysis is that $a_2^{\rm eff}$ is negative in charm decay, whereas it becomes positive in the two-body decays of the B meson [14,17,7]:

$$a_2^{\rm eff}(D\to\bar{K}\pi)\sim-0.50,\quad a_2^{\rm eff}(B\to D\pi)\sim0.20-0.28. \tag{7}$$

It should be stressed that the magnitude of $a_{1,2}$ depends on the model results for form factors. It follows that

$$\chi_2(D\to\bar{K}\pi)\sim-0.36,\quad \chi_2(B\to D\pi)\sim0.12-0.19. \tag{8}$$

The observation $|\chi_2(B)|\ll|\chi_2(D)|$ is consistent with the intuitive picture that soft gluon effects become stronger when the final-state particles move slower, allowing more time for significant final-state interactions after hadronization [14]. Phenomenologically, it is often to treat the number of colors N_c as a free parameter to model the nonfactorizable contribution to hadronic matrix elements and its value can be extracted from the data of two-body nonleptonic decays. Theoretically, this amounts to defining an effective number of colors $N_c^{\rm eff}$, called $1/\xi$ in Ref. [18], by

$$1/N_c^{\rm eff}\equiv(1/N_c)+\chi. \tag{9}$$

It is clear from Eq. (8) that

$$N_c^{\rm eff}(D\to\bar{K}\pi)\gg3,\quad N_c^{\rm eff}(B\to D\pi)\sim1.8-2.2. \tag{10}$$

The effective Wilson coefficients appear in the factorizable decay amplitudes in the combinations $a_{2i}=c_{2i}^{\rm eff}+(1/N_c)c_{2i-1}^{\rm eff}$ and $a_{2i-1}=c_{2i-1}^{\rm eff}+(1/N_c)c_{2i}^{\rm eff}$ $(i=1,\dots,5)$. As discussed in the Introduction, nonfactorizable effects in the decay amplitudes of $B\to PP,\ VP$ can be absorbed into the parameters $a_i^{\rm eff}$. This amounts to replacing N_c in $a_i^{\rm eff}$ by $(N_c^{\rm eff})_i$. Explicitly,

$$a_{2i}^{\rm eff}=c_{2i}^{\rm eff}+\frac{1}{(N_c^{\rm eff})_{2i}}c_{2i-1}^{\rm eff},$$

$$a_{2i-1}^{\rm eff}=c_{2i-1}^{\rm eff}+\frac{1}{(N_c^{\rm eff})_{2i-1}}c_{2i}^{\rm eff}\quad(i=1,\dots,5). \tag{11}$$

It is customary to assume in the literature that $(N_c^{\rm eff})_1\approx(N_c^{\rm eff})_2\cdots\approx(N_c^{\rm eff})_{10}$ so that the subscript i can be dropped; that is, the nonfactorizable term is usually assumed to behave in the same way in penguin and tree decay amplitudes. A closer investigation shows that this is not the case. We have argued in Ref. [12] that nonfactorizable effects in the matrix elements of $(V-A)(V+A)$ operators are *a priori* different from that of $(V-A)(V-A)$ operators. One reason is that the Fierz transformation of the $(V-A)(V+A)$ operators $O_{5,6,7,8}$ is quite different from that of $(V-A)(V-A)$ operators $O_{1,2,3,4}$ and $O_{9,10}$. As a result, contrary to the common assumption, $N_c^{\rm eff}(LR)$ induced by the $(V-A)(V+A)$ operators are theoretically different from $N_c^{\rm eff}(LL)$ generated by the $(V-A)(V-A)$ operators [12]. From Eq. (11) it is expected that

$$N_c^{\rm eff}(LL)\equiv(N_c^{\rm eff})_1\approx(N_c^{\rm eff})_2\approx(N_c^{\rm eff})_3\approx(N_c^{\rm eff})_4\approx(N_c^{\rm eff})_9\approx(N_c^{\rm eff})_{10},$$

$$N_c^{\rm eff}(LR)\equiv(N_c^{\rm eff})_5\approx(N_c^{\rm eff})_6\approx(N_c^{\rm eff})_7\approx(N_c^{\rm eff})_8, \tag{12}$$

and $N_c^{\rm eff}(LR)\neq N_c^{\rm eff}(LL)$ in general. In principle, $N_c^{\rm eff}$ can vary from channel to channel, as in the case of charm decay. However, in the energetic two-body B decays, $N_c^{\rm eff}$ is expected to be process insensitive as supported by data [7].

TABLE I. Numerical values for the effective coefficients $a_i^{\rm eff}$ at $N_c^{\rm eff}=2,3,5,\infty$ (in units of 10^{-4} for $a_3,\dots,a_{10}$). For simplicity we will drop the superscript eff henceforth.

	$N_c^{\rm eff}=2$	$N_c^{\rm eff}=3$	$N_c^{\rm eff}=5$	$N_c^{\rm eff}=\infty$
a_1	0.986	1.04	1.08	1.15
a_2	0.25	0.058	–0.095	–0.325
a_3	$-13.9-22.6i$	61	$121+18.1i$	$211+45.3i$
a_4	$-344-113i$	$-380-121i$	$-408-127i$	$-450-136i$
a_5	$-146-22.6i$	-52.7	$22.0+18.1i$	$134+45.3i$
a_6	$-493-113i$	$-515-121i$	$-533-127i$	$-560-136i$
a_7	$0.04-2.73i$	$-0.71-2.73i$	$-1.24-2.73i$	$-2.04-2.73i$
a_8	$2.98-1.37i$	$3.32-0.91i$	$3.59-0.55i$	4
a_9	$-87.9-2.73i$	$-91.1-2.73i$	$-93.7-2.73i$	$-97.6-2.73i$
a_{10}	$-29.3-1.37i$	$-13.1-0.91i$	$-0.04-0.55i$	19.48

The $N_c^{\rm eff}$ dependence of the effective parameters $a_i^{\rm eff}$'s are shown in Table I for several representative values of $N_c^{\rm eff}$. From Table I we see that (i) the dominant coefficients are $a_1,\ a_2$ for current-current amplitudes, a_4 and a_6 for QCD penguin-induced amplitudes, and a_9 for electroweak penguin-induced amplitudes, and (ii) a_1,a_4,a_6, and a_9 are $N_c^{\rm eff}$ stable, while others depend strongly on $N_c^{\rm eff}$. Therefore, for charmless B decays whose decay amplitudes depend dominantly on $N_c^{\rm eff}$-stable coefficients, their decay rates can be reliably predicted within the factorization approach even in the absence of information on nonfactorizable effects.

The CLEO data of $B^\pm\to\omega\pi^\pm$ available last year clearly indicate that $N_c^{\rm eff}(LL)$ is favored to be small, $N_c^{\rm eff}(LL)<2.9$ [12]. If the value of $N_c^{\rm eff}(LL)$ is fixed to be 2, the branching ratio of $B^\pm\to\omega\pi^\pm$ for positive ρ (ρ being a Wolfenstein parameter; see Sec. II D), which is preferred by the current analysis [19], will be of order $(0.9-1.0)\times10^{-5}$, which is very close to the central value of the measured one. Unfortunately, the significance of $B^\pm\to\omega\pi^\pm$ is reduced in the recent CLEO analysis and only an upper limit is quoted [20]. Nevertheless, the central value of $\mathcal{B}(B^\pm\to\pi^\pm\omega)$ remains about the same. Therefore, a measurement of its branching ratio is urgently needed. A very recent CLEO analysis of $B^0\to\pi^+\pi^-$ [21] presents an improved upper limit, $\mathcal{B}(B^0\to\pi^+\pi^-)<0.84\times10^{-5}$. If the form factor $F_0^{B\pi}(0)$ is known, this tree-dominated decay could offer a useful constraint on $N_c^{\rm eff}(LL)$ as its branching ratio increases slightly with $N_c^{\rm eff}$. For $F_0^{B\pi}(0)=0.30$, we find $N_c^{\rm eff}(LL)\lesssim 2.0$. The fact that $N_c^{\rm eff}(LL)$ is favored to be at the value of 2 in hadronic charmless two-body decays of the B meson is consistent with the nonfactorizable term extracted from $B\to(D,D^*)\pi$, $D\rho$ decays, namely, $N_c^{\rm eff}(B\to D\pi)\approx 2$. Since the energy release in the energetic two-body decays $B\to\omega\pi, B\to D\pi$ is of the same order of magnitude, it is thus expected that $N_c^{\rm eff}(LL)|_{B\to\omega\pi}\approx 2$. In analogue to the class-III $B\to D\pi$ decays, the interference effect of spectator amplitudes in charged B decays $B^-\to\pi^-\pi^0$, $\rho^-\pi^0$, $\pi^-\rho^0$ is sensitive to $N_c^{\rm eff}(LL)$; measurements of them will be very useful to pin down the value of $N_c^{\rm eff}(LL)$.

As for $N_c^{\rm eff}(LR)$, it is found in Ref. [12] that the constraints on $N_c^{\rm eff}(LR)$ derived from $B^\pm\to\phi K^\pm$ and $B\to\phi K^*$ are not consistent. Under the factorization hypothesis, the decays $B\to\phi K$ and $B\to\phi K^*$ should have almost the same branching ratios, a prediction not borne out by current data. Therefore, it is crucial to measure the charged and neutral decay modes of $B\to\phi(K,K^*)$ in order to see if the generalized factorization approach is applicable to $B\to\phi K^*$ decay. Nevertheless, the analysis of $B\to\eta' K$ in Ref. [12] indicates that $N_c^{\rm eff}(LL)\approx 2$ is favored and $N_c^{\rm eff}(LR)$ is preferred to be larger. Since the energy release in the energetic two-body charmless B decays is not less than that in $B\to D\pi$ decays, it is thus expected that

$$|\chi(\text{two-body rare B decay})|\lesssim|\chi(B\to D\pi)|. \tag{13}$$

It follows from Eqs. (8) and (9) that $N_c^{\rm eff}(LL)\approx N_c^{\rm eff}(B\to D\pi)\sim 2$ and $N_c^{\rm eff}(LR)\sim 2-5$, depending on the sign of χ. Since $N_c^{\rm eff}(LR)>N_c^{\rm eff}(LL)$ implied by the data, therefore, we conjecture that

$$N_c^{\rm eff}(LL)\approx 2, \quad N_c^{\rm eff}(LR)\lesssim 5. \tag{14}$$

C. Factorizable amplitudes and their classification

Applying the effective Hamiltonian (2), the factorizable decay amplitudes of $\overline{B}_s\to PP,VP,VV$ obtained within the generalized factorization approach are summarized in Appendixes A,B,C, where, for simplicity, we have neglected W annihilation, spacelike penguins, and final-state interactions. All the penguin contributions to the decay amplitudes can be derived from Table II by studying the underlying b quark weak transitions. To illustrate this, let $X^{(BM_1,M_2)}$ denote the factorizable amplitude with the meson M_2 being factored out:

$$X^{(BM_1,M_2)}=\langle M_2|(\overline{q}_2q_3)_{V-A}|0\rangle\langle M_1|(\overline{q}_1b)_{V-A}|\overline{B}\rangle. \tag{15}$$

In general, when M_2 is a charged state, only $a_{\rm even}$ penguin terms contribute. For example, from Table II we obtain

TABLE II. Penguin contributions to the factorizable $B\to PP$, VP,VV decay amplitudes multiplied by $-(G_F/\sqrt{2})V_{tb}V_{tq}^*$, where $q=d,s$. The notation $B\to M_1,M_2$ means that the meson M_2 can be factored out under the factorizable approximation. In addition to the $a_{\rm even}$ terms, the decay also receives contributions from $a_{\rm odd}$ penguin effects when M_2 is a neutral meson with $I_3=0$. Except for η or η' production, the coefficients R and R' are given by $R=2m_P^2/[(m_1+m_2)(m_b-m_3)]$ and $R'=-2m_P^2/[(m_1+m_2)(m_b+m_3)]$, respectively.

Decay	$b\to qu\overline{u},\ b\to qc\overline{c}$	$b\to qd\overline{d},\ b\to qs\overline{s}$
$B\to P,P$	$a_4+a_{10}+(a_6+a_8)R$	$a_4-\frac{1}{2}a_{10}+(a_6-\frac{1}{2}a_8)R$
$B\to V,P$	$a_4+a_{10}+(a_6+a_8)R'$	$a_4-\frac{1}{2}a_{10}+(a_6-\frac{1}{2}a_8)R'$
$B\to P,V$	a_4+a_{10}	$a_4-\frac{1}{2}a_{10}$
$B\to V,V$	a_4+a_{10}	$a_4-\frac{1}{2}a_{10}$
$B\to P,P^0$	$a_3-a_5-a_7+a_9$	$a_3-a_5+\frac{1}{2}a_7-\frac{1}{2}a_9$
$B\to V,P^0$	$a_3-a_5-a_7+a_9$	$a_3-a_5+\frac{1}{2}a_7-\frac{1}{2}a_9$
$B\to P,V^0$	$a_3+a_5+a_7+a_9$	$a_3+a_5-\frac{1}{2}a_7-\frac{1}{2}a_9$
$B\to V,V^0$	$a_3+a_5+a_7+a_9$	$a_3+a_5-\frac{1}{2}a_7-\frac{1}{2}a_9$

$$A(\overline{B}_s\to K^+\pi^-)_{\rm peng}\propto[a_4+a_{10}+(a_6+a_8)R]X^{(B_sK^+,\pi^-)},$$

$$A(\overline{B}_s\to K^{*+}\pi^-)_{\rm peng}\propto[a_4+a_{10}-(a_6+a_8)R']X^{(B_sK^{*+},\pi^-)},$$

$$A(\overline{B}_s\to K^+\rho^-)_{\rm peng}\propto[a_4+a_{10}]X^{(B_sK^+,\rho^-)}, \tag{16}$$

with $R'\approx R\approx m_\pi^2/(m_bm_d)$. When M_2 is a neutral meson with $I_3=0$, namely, $M_2=\pi^0,\rho^0,\omega$ and $\eta^{(\prime)}$,$a_{\rm odd}$ penguin terms start to contribute. From Table II we see that the decay amplitudes of $\overline{B}_s\to M\pi^0$, $\overline{B}_s\to M\rho^0$, $\overline{B}_s\to M\omega$, $\overline{B}_s\to M\eta^{(\prime)}$ contain the following respective factorizable terms:

$$\frac{3}{2}(-a_7+a_9)X_u^{(B_sM,\pi^0)},$$

$$\frac{3}{2}(a_7+a_9)X_u^{(B_sM,\rho^0)},$$

$$\left(2a_3+2a_5+\frac{1}{2}a_7+\frac{1}{2}a_9\right)X_u^{(B_sM,\omega)},$$

$$\left(2a_3-2a_5-\frac{1}{2}a_7+\frac{1}{2}a_9\right)X_u^{(B_sM,\eta^{(\prime)})}, \tag{17}$$

where the subscript u indicates the $u\overline{u}$ quark content of the neutral meson:

$$X_u^{(B_sM,\pi^0)}=\langle\pi^0|(\overline{u}u)_{V-A}|0\rangle\langle M_1|(\overline{q}_1b)_{V-A}|\overline{B}_s\rangle. \tag{18}$$

For example, the penguin amplitudes of $\overline{B}_s\to\eta\omega$ and $K^0\pi^0$ are given by

$$A(\overline{B}_s\to\eta\omega)_{\rm peng}\propto\left[2(a_3+a_5)+\frac{1}{2}(a_7+a_9)\right]X_u^{(B_s\eta,\omega)},$$

$$\begin{aligned}A(\overline{B}_s\to K^0\pi^0)_{\rm peng}\propto&\frac{3}{2}(-a_7+a_9)X_u^{(B_sK^0,\pi^0)}\\&+\left[a_4-\frac{1}{2}a_{10}+\left(a_6-\frac{1}{2}a_8\right)R\right]X_d^{(B_sK^0,\pi^0)}\\\propto&\left[-a_4+\frac{3}{2}(-a_7+a_9)+\frac{1}{2}a_{10}\right.\\&\left.-\left(a_6-\frac{1}{2}a_8\right)R\right]X_u^{(B_sK^0,\pi^0)},\end{aligned}\tag{19}$$

respectively. It is interesting to note that the decays $\overline{B}_s\to(\eta^{(\prime)},\phi)(\pi^0,\rho^0)$ do not receive any contributions from QCD penguin diagrams and they are dominated by electroweak penguins. We will come back to this interesting observation later.

Just as the charm decays or B decays into the charmed meson, the tree-dominated amplitudes for hadronic charmless B decays are customarily classified into three classes [18]

Class I for the decay modes dominated by the external W emission characterized by the parameter a_1. Examples are $\overline{B}_s\to K^+\pi^-$, $K^{*+}\pi^-$, ...

Class II for the decay modes dominated by the color-suppressed internal W emission characterized by the parameter a_2. Examples are $\overline{B}_s\to K^0\pi^0$, $K^0\rho^0$, ...

Class III decays involving both external and internal W emissions. Hence the class-III amplitude is of the form a_1+ra_2. This class does not exist for the B_s.

Likewise, penguin-dominated charmless B_s decays can be classified into three categories.[2]

Class IV for those decays whose amplitudes are governed by the QCD penguin parameters a_4 and a_6 in the combination a_4+Ra_6, where the coefficient R arises from the $(S-P)(S+P)$ part of the operator O_6. In general, $R=2m_{P_b}^2/[(m_1+m_2)(m_b-m_3)]$ for $B\to P_aP_b$ with the meson P_b being factored out under the factorizable approximation, $R=-2m_{P_b}^2/[(m_1+m_2)(m_b+m_3)]$ for $B\to V_aP_b$, and $R=0$ for $B\to P_aV_b$ and $B\to V_aV_b$. Note that a_4 is always accompanied by a_{10}, and a_6 by a_8. In short, class-IV modes are governed by $a_{\rm even}$ penguin terms. Examples are $\overline{B}_s\to K^+K^-$, $K^0\overline{K}^0$, $\phi\eta^{(\prime)}$, ...

Class V modes for those decays whose amplitudes are governed by the effective coefficients a_3,a_5,a_7, and a_9 (i.e., $a_{\rm odd}$ penguin terms) in the combinations $a_3\pm a_5$ and/or $a_7\pm a_9$ (see Table II). Examples are $\overline{B}_s\to\pi\eta^{(\prime)}$, $\omega\eta^{(\prime)}$, $\pi\phi$, ...

Class-VI involving the interference of class-IV and class-V decays, e.g., $\overline{B}_s\to\eta^{(\prime)}\eta^{(\prime)},\phi\eta^{(\prime)},K^0\phi$, ...

Sometimes the tree and penguin contributions are comparable. In this case, the interference between penguin and spectator amplitudes is at work. There are three such decays: $\overline{B}_s\to K^0\omega,K^{*0}\eta^{(\prime)},K^{*0}\omega$; they involve class-II and -VI amplitudes (see Tables IV and V).

D. Input parameters

In this subsection we specify the values for various parameters employed in the present paper. For current quark masses, we employ the running masses at the scale $\mu=m_b$:

$$\begin{aligned}&m_u(m_b)=3.2\ \text{MeV},\quad m_d(m_b)=6.4\ \text{MeV},\\&m_s(m_b)=105\ \text{MeV},\\&m_c(m_b)=0.95\ \text{GeV},\quad m_b(m_b)=4.34\ \text{GeV}.\end{aligned}\tag{20}$$

As for the Wolfenstein parameters A,λ,ρ, and η, which are utilized to parametrize the quark mixing matrix, we use $A=0.804$, $\lambda=0.22$, $\rho=0.16$, and $\eta=0.34$. The values for ρ and η follow from a recent analysis of all available experimental constraints imposed on the Wolfenstein parameters [19]:

$$\overline{\rho}=0.156\pm0.090,\quad\overline{\eta}=0.328\pm0.054,\tag{21}$$

where $\overline{\rho}=\rho(1-\lambda^2/2)$ and $\overline{\eta}=\eta(1-\lambda^2/2)$. For the values of decay constants, we use $f_\pi=132$ MeV, $f_K=160$ MeV, $f_\rho=210$ MeV, $f_{K^*}=221$ MeV, $f_\omega=195$ MeV, and $f_\phi=237$ MeV.

To determine the decay constant $f^q_{\eta^{(\prime)}}$, defined by $\langle 0|\overline{q}\gamma_\mu\gamma_5 q|\eta^{(\prime)}\rangle=if^q_{\eta^{(\prime)}}p_\mu$, it has been emphasized [24,25] that the decay constants do not simply follow the $\eta-\eta'$ state mixing given by

$$\eta'=\eta_8\sin\theta+\eta_0\cos\theta,\quad\eta=\eta_8\cos\theta-\eta_0\sin\theta.\tag{22}$$

Introduce the decay constants f_8 and f_0 by

$$\langle 0|A^0_\mu|\eta_0\rangle=if_0p_\mu,\quad\langle 0|A^8_\mu|\eta_8\rangle=if_8p_\mu.\tag{23}$$

Because of SU(3) breaking, the matrix elements $\langle 0|A_\mu^{0(8)}|\eta_{8(0)}\rangle$ do not vanish in general and they will induce a two-angle mixing among the decay constants, that is, $f^u_{\eta'}$ and $f^s_{\eta'}$ are related to f_8 and f_0 by

$$\begin{aligned}f^u_{\eta'}&=\frac{f_8}{\sqrt6}\sin\theta_8+\frac{f_0}{\sqrt3}\cos\theta_0,\\f^s_{\eta'}&=-2\frac{f_8}{\sqrt6}\sin\theta_8+\frac{f_0}{\sqrt3}\cos\theta_0.\end{aligned}\tag{24}$$

Likewise,

[2]Our classification of factorizable penguin amplitudes is not the same as that in Ref. [22]; we introduce three new classes in the same spirit as the classification of tree-dominated decays.

$$f_\eta^u = \frac{f_8}{\sqrt{6}}\cos\theta_8 - \frac{f_0}{\sqrt{3}}\sin\theta_0,$$

$$f_\eta^s = -2\frac{f_8}{\sqrt{6}}\cos\theta_8 - \frac{f_0}{\sqrt{3}}\sin\theta_0. \tag{25}$$

Based on the ansatz that the decay constants in the quark flavor basis follow the pattern of particle state mixing, relations between θ_8, θ_0 and θ are derived in Ref. [25], where θ is the $\eta-\eta'$ mixing angle introduced in Eq. (22). It is found in Ref. [25] that phenomenologically

$$\theta_8 = -21.2°, \quad \theta_0 = -9.2°, \quad \theta = -15.4°, \tag{26}$$

and

$$f_8/f_\pi = 1.26, \quad f_0/f_\pi = 1.17. \tag{27}$$

The decay constant $f_{\eta'}^c$, defined by $\langle 0|\bar{c}\gamma_\mu\gamma_5 c|\eta'\rangle = i f_{\eta'}^c q_\mu$, has been determined from theoretical calculations [27–29] and from the phenomenological analysis of the data of $J/\psi\to\eta_c\gamma$, $J/\psi\to\eta'\gamma$ and of the $\eta\gamma$ and $\eta'\gamma$ transition form factors [11,25,30–32]; it lies in the range -2.3 MeV $\leq f_{\eta'}^c \leq -18.4$ MeV. In this paper we use the values

$$f_{\eta'}^c = -(6.3\pm0.6)\ \text{MeV}, \quad f_\eta^c = -(2.4\pm0.2)\ \text{MeV}, \tag{28}$$

as obtained in Ref. [25].

For form factors, the Bauer-Stech-Wirbel (BSW) model [26] gives [5][3]

$$F_0^{B_sK}(0) = 0.274, \quad F_0^{B_s\eta_{s\bar{s}}}(0) = 0.335, \quad F_0^{B_s\eta'_{s\bar{s}}}(0) = 0.282,$$

$$A_0^{B_s\phi}(0) = 0.272, \quad A_1^{B_s\phi}(0) = 0.273, \quad A_2^{B_s\phi}(0) = 0.273,$$

$$A_0^{B_sK^*}(0) = 0.236, \quad A_1^{B_sK^*}(0) = 0.232, \quad A_2^{B_sK^*}(0) = 0.231,$$

$$V^{B_s\phi}(0) = 0.319, \quad V^{B_sK^*}(0) = 0.281. \tag{29}$$

It should be stressed that the $\eta-\eta'$ wave function normalization has not been included in the form factors $F_0^{B_s\eta_{s\bar{s}}}$ and $F_0^{B_s\eta'_{s\bar{s}}}$; they are calculated in a relativistic quark model by putting the $s\bar{s}$ constitutent quark mass only. To compute the physical form factors, one has to take into account the wave function normalizations of η and η':

$$F_0^{B_s\eta} = -\left(\frac{2}{\sqrt{6}}\cos\theta + \frac{1}{\sqrt{3}}\sin\theta\right)F_0^{B_s\eta_{s\bar{s}}},$$

$$F_0^{B_s\eta'} = \left(-\frac{2}{\sqrt{6}}\sin\theta + \frac{1}{\sqrt{3}}\cos\theta\right)F_0^{B_s\eta'_{s\bar{s}}}. \tag{30}$$

It is clear that the form factors $F_0^{B_s\eta}$ and $F_0^{B_s\eta'}$ have opposite signs.

For the q^2 dependence of form factors in the region where q^2 is not too large, we shall use the pole dominance ansatz, namely,

$$f(q^2) = \frac{f(0)}{(1-q^2/m_*^2)^n}, \tag{31}$$

where m_* is the pole mass given in Ref. [18]. A direct calculation of $B\to P$ and $B\to V$ form factors at timelike momentum transfers is available in the relativistic light-front quark model [33] with the results that the q^2 dependence of the form factors A_0, A_2, V, F_1 is a dipole behavior (i.e., $n=2$), while F_0, A_1 exhibit a monopole dependence ($n=1$).

Recently, the $B_s\to K^*$ and $B_s\to\phi$ form factors have also been calculated in the light-cone sum rule approach [34] with the parametrization

$$f(q^2) = \frac{f(0)}{1-a(q^2/m_{B_s}^2)+b(q^2/m_{B_s}^2)^2} \tag{32}$$

for the form-factor q^2 dependence. The results are [34]

$$A_0^{B_s\phi}(0) = 0.382, \quad a = 1.77, \quad b = 0.856,$$

$$A_1^{B_s\phi}(0) = 0.296, \quad a = 0.87, \quad b = -0.061,$$

$$A_2^{B_s\phi}(0) = 0.255, \quad a = 1.55, \quad b = 0.513,$$

$$V^{B_s\phi}(0) = 0.433, \quad a = 1.75, \quad b = 0.736,$$

$$A_0^{B_sK^*}(0) = 0.254, \quad a = 1.87, \quad b = 0.887,$$

$$A_1^{B_sK^*}(0) = 0.190, \quad a = 1.02, \quad b = -0.037,$$

$$A_2^{B_sK^*}(0) = 0.164, \quad a = 1.77, \quad b = 0.729,$$

$$V^{B_sK^*}(0) = 0.262, \quad a = 1.89, \quad b = 0.846. \tag{33}$$

It is obvious that the q^2 dependence for the form factors A_0, A_2, and V is dominated by the dipole terms, while A_1 by the monopole term in the region where q^2 is not too large. In Tables IV and V we will present results using these two different parametrizations for $B_s\to V$ form factors.

We will encounter matrix elements of pseudoscalar densities when evaluating the penguin amplitudes. Care must be taken to consider the pseudoscalar matrix element for $\eta^{(\prime)}\to$ vacuum transition: The anomaly effects must be included in order to ensure a correct chiral behavior for the pseudoscalar matrix element [12]. The results are [35,11]

[3]The form factors adopted in Ref. [6] are calculated using the light-front quark model and in general they are larger than the BSW model's results.

$$\langle \eta^{(\prime)}|\bar{s}\gamma_5 s|0\rangle = -i\frac{m^2_{\eta^{(\prime)}}}{2m_s}(f^s_{\eta^{(\prime)}} - f^u_{\eta^{(\prime)}}),$$

$$\langle \eta^{(\prime)}|\bar{u}\gamma_5 u|0\rangle = \langle \eta^{(\prime)}|\bar{d}\gamma_5 d|0\rangle = r_{\eta^{(\prime)}}\langle \eta^{(\prime)}|\bar{s}\gamma_5 s|0\rangle, \tag{34}$$

with [12]

$$r_{\eta'} = \frac{\sqrt{2f_0^2 - f_8^2}}{\sqrt{2f_8^2 - f_0^2}}\frac{\cos\theta + \frac{1}{\sqrt{2}}\sin\theta}{\cos\theta - \sqrt{2}\sin\theta},$$

$$r_{\eta} = -\frac{1}{2}\frac{\sqrt{2f_0^2 - f_8^2}}{\sqrt{2f_8^2 - f_0^2}}\frac{\cos\theta - \sqrt{2}\sin\theta}{\cos\theta + \frac{1}{\sqrt{2}}\sin\theta}. \tag{35}$$

III. NUMERICAL RESULTS AND DISCUSSIONS

With the factorizable decay amplitudes summarized in Appendixes and the input parameters shown in Sec. II, we are ready to compute the branching ratios for the two-body charmless nonleptonic decays of the B_s meson. The decay rates for $B_s \to PP, VP$ are given by

$$\Gamma(B_s \to P_1P_2) = \frac{p_c}{8\pi m^2_{B_s}}|A(B_s \to P_1P_2)|^2,$$

$$\Gamma(B_s \to VP) = \frac{p_c^3}{8\pi m_V^2}|A(B_s \to VP)/(\varepsilon \cdot p_{B_s})|^2. \tag{36}$$

The decay $B_s \to VV$ is more complicated as its amplitude involves three form factors. In general, the factorizable amplitude of $B_s \to V_1V_2$ is of the form

$$\begin{aligned} A(B_s \to V_1V_2) &= \alpha X^{(B_sV_1,V_2)} + \beta X^{(B_sV_2,V_1)} \\ &= (\alpha_1 A_1^{B_sV_1} + \beta_1 A_1^{B_sV_2})\varepsilon_1^* \cdot \varepsilon_2^* \\ &\quad + (\alpha_2 A_2^{B_sV_1} + \beta_2 A_2^{B_sV_2})(\varepsilon_1^* \cdot p_{B_s})(\varepsilon_2^* \cdot p_{B_s}) \\ &\quad + i\varepsilon_{\mu\nu\rho\sigma}\varepsilon_2^{*\mu}\varepsilon_1^{*\nu}p^\rho_{B_s}p_1^\sigma(\alpha_3 V^{B_sV_1} \\ &\quad + \beta_3 V^{B_sV_2}), \end{aligned} \tag{37}$$

where use of Eq. (C1) has been made. Then

$$\Gamma(B_s \to V_1V_2) = \frac{p_c}{8\pi m^2_{B_s}}|\alpha_1(m_{B_s}+m_1)m_2 f_{V_2}A_1^{B_sV_1}(m_2^2)|^2 \times (H + 2\zeta H_1 + 2\zeta^2 H_2), \tag{38}$$

where

$$H = (a - bx)^2 + 2(1 + c^2y^2),$$

$$H_1 = (a - bx)(a - b'x') + 2(1 + cc'yy'),$$

$$H_2 = (a - b'x')^2 + 2(1 + c'^2y'^2), \tag{39}$$

with

$$a = \frac{m^2_{B_s} - m_1^2 - m_2^2}{2m_1m_2}, \quad b = \frac{2m^2_{B_s}p_c^2}{m_1m_2(m_{B_s}+m_1)^2},$$

$$c = \frac{2m_{B_s}p_c}{(m_{B_s}+m_1)^2},$$

$$\zeta = \frac{\beta_1 A_1^{B_sV_2}(m_1^2)}{\alpha_1 A_1^{B_sV_1}(m_2^2)}, \quad x = \frac{A_2^{B_sV_1}(m_2^2)}{A_1^{B_sV_1}(m_2^2)}, \quad y = \frac{V^{B_sV_1}(m_2^2)}{A_1^{B_sV_1}(m_2^2)}, \tag{40}$$

where p_c is the c.m. momentum, $m_1(m_2)$ is the mass of the vector meson $V_1(V_2)$, and b', c', x', y' can be obtained from b, c, x, y, respectively, with the replacement $V_1 \leftrightarrow V_2$.

The calculated branching ratios for $B_s \to PP, VP, VV$ decays averaged over CP-conjugate modes are shown in Tables III–V, respectively, where the nonfactorizable effects are treated in two different cases: (i) $N_c^{\rm eff}(LL) \neq N_c^{\rm eff}(LR)$ with the former being fixed at the value of 2, and (ii) $N_c^{\rm eff}(LL) = N_c^{\rm eff}(LR)$. For decay modes involving $B_s \to K^*$ or $B_s \to \phi$ transition, we apply two different models for form factors: the BSW model [see Eq. (29)] and the light-cone sum rule approach [see Eq. (33)]. To compute the branching ratio, we have used the B_s lifetime [3]

$$\tau(B_s) = (1.54 \pm 0.07) \times 10^{-12} \ \text{s}. \tag{41}$$

From Tables III–V we see that the branching ratios for class-I and -IV modes are stable against the variation of $N_c^{\rm eff}$ as they depend on the coefficients a_1, a_4 and a_6 which are $N_c^{\rm eff}$-stable. Class-V channels in general depend on the coefficients $a_3 + a_5$ and $a_7 + a_9$. However, the decays

$$\bar{B}_s \to \eta\pi, \ \ \eta'\pi, \ \ \eta\rho, \ \ \eta'\rho, \ \ \phi\pi, \ \ \phi\rho \tag{42}$$

do not receive any QCD penguin contributions [2]. Therefore, these six decay modes are predominantly governed by the electroweak penguin coefficient a_9, which is $N_c^{\rm eff}$-insensitive. A measurement of them can be utilized to fix the parameter a_9. Note that their branching ratios are in general small, ranging from 4×10^{-8} to 0.4×10^{-6}, but they could be accessible at the future hadron colliders with large b production.

In order to see the relative importance of electroweak penguin effects in penguin-dominated B_s decays, we follow Ref. [22] to compute the ratio

$$R_W = \frac{\mathcal{B}(B_s \to h_1h_2)(\text{with } \ a_7, \ldots, a_{10} = 0)}{\mathcal{B}(B_s \to h_1h_2)}. \tag{43}$$

Obviously, if the tree, QCD penguin and electroweak penguin amplitudes are of the same sign, then $(1 - R_W)$ mea-

TABLE III. Branching ratios (in units of 10^{-6}) averaged over CP-conjugate modes for charmless $\overline{B}_s \to PP$ decays. Predictions are for $k^2=m_b^2/2$, $\eta=0.34$, $\rho=0.16$, and $N_c^{\rm eff}(LR)=2,3,5,\infty$ with $N_c^{\rm eff}(LL)$ being fixed to be 2 in the first case and treated to be the same as $N_c^{\rm eff}(LR)$ in the second case. We use the BSW model for form factors [see Eq. (29)].

Decay	Class	$N_c^{\rm eff}(LL)=2$				$N_c^{\rm eff}(LL)=N_c^{\rm eff}(LR)$			
		2	3	5	∞	2	3	5	∞
$\overline{B}_s\to K^+\pi^-$	I	6.64	6.66	6.67	6.70	6.64	7.38	8.01	8.99
$\overline{B}_s\to K^0\pi^0$	II	0.24	0.24	0.25	0.25	0.24	0.08	0.12	0.46
$\overline{B}_s\to K^+K^-$	IV	9.88	10.9	10.9	11.6	9.88	10.9	11.7	12.9
$\overline{B}_s\to K^0\overline{K}^0$	IV	10.3	10.9	11.4	12.1	10.3	12.0	13.5	15.8
$\overline{B}_s\to \pi^0\eta'$	V	0.04	0.04	0.04	0.04	0.04	0.05	0.06	0.09
$\overline{B}_s\to \pi^0\eta$	V	0.04	0.04	0.04	0.04	0.04	0.05	0.06	0.09
$\overline{B}_s\to K^0\eta'$	VI	0.63	0.86	1.06	1.42	0.63	0.54	0.57	0.76
$\overline{B}_s\to K^0\eta$	VI	0.81	0.84	0.87	0.91	0.81	0.82	0.96	1.39
$\overline{B}_s\to \eta\eta'$	VI	12.5	16.3	19.6	25.3	12.5	14.4	15.9	18.5
$\overline{B}_s\to \eta'\eta'$	VI	6.28	10.3	14.3	21.4	6.28	6.80	7.23	7.91
$\overline{B}_s\to \eta\eta$	VI	5.30	4.80	4.41	3.89	5.30	6.23	7.05	8.37

sures the fraction of non-electroweak penguin contributions to $\mathcal{B}(B_s\to h_1h_2)$. It is evident from Table VI that the decays listed in Eq. (42) all have the same $N_c^{\rm eff}$ dependence: For $N_c^{\rm eff}(LL)=2$, the electroweak penguin contributions account for 85% of the branching ratios for $B_s\to\eta\pi,\ldots,\phi\rho$, and the ratio R_W is very sensitive to $N_c^{\rm eff}$ when $N_c^{\rm eff}(LL)=N_c^{\rm eff}(LR)$. We also see that electroweak penguin corrections to

$$\overline{B}_s\to\omega\eta,\ \ \omega\eta',\ \ \phi\eta,\ \ \phi\eta',\ \ \omega\phi,\ \ K\phi,\ \ K^*\phi,\ \ \phi\phi, \tag{44}$$

depending very sensitively on $N_c^{\rm eff}$, are in general as important as QCD penguin effects and even play a dominant role. For example, about 50% of $\mathcal{B}(\overline{B}_s\to K^0\phi)$ comes from the electroweak penguin contributions at $N_c^{\rm eff}(LL)=2$ and $N_c^{\rm eff}(LR)=5$.

Strictly speaking, because of variously possible interference of the electroweak penguin amplitude with the tree and QCD penguin contributions, R_W is not the most suitable quantity for measuring the relative importance of electroweak penguin effects. For example, it appears at the first sight that only 21% of $\mathcal{B}(B_s\to\omega\eta')$ and $\mathcal{B}(B_s\to\omega\phi)$ arises from the electroweak penguins at $N_c^{\rm eff}(LL)=N_c^{\rm eff}(LR)=3$. However, the decay amplitudes are proportional to (see Appendix B)

$$V_{ub}V_{us}^*a_2-V_{tb}V_{ts}^*\left[2(a_3+a_5)+\frac{1}{2}(a_7+a_9)\right]. \tag{45}$$

Since a_2 and (a_3+a_5) are minimum at $N_c^{\rm eff}\sim3$ (see Table I), the decay is obviously dominated by the electroweak penguin transition when $N_c^{\rm eff}(LL)=N_c^{\rm eff}(LR)=3$. Numerically, we find at the amplitude level

$$\text{tree: QCDpenguin: electroweak penguin}=0.28:1:-2.72. \tag{46}$$

It is clear that although $R_W=0.79$ for $N_c^{\rm eff}=3$, the decays $B_s\to\omega\eta'$ and $B_s\to\omega\phi$ are actually dominated by the electroweak penguin.

The branching ratios for the class-V and -VI modes shown in Eq. (44) depend strongly on the value of $N_c^{\rm eff}$. As pointed out in Sec. II, the preferred values for the effective number of colors are $N_c^{\rm eff}(LL)\approx2$ and $N_c^{\rm eff}(LR)\sim5$. We believe that the former will be confirmed soon by the forthcoming measurements of $B\to\pi\pi,\ \pi\rho,\ldots$. However, the branching ratios for some of the decay modes, e.g. $B_s\to\omega\eta,\omega\eta',\phi\eta$, become very small at the values of $N_c^{\rm eff}$ given by Eq. (14). As suggested in Ref. [22], these decays involve large cancellation among competing amplitudes and they may receive significant contributions from annihilation and/or final-state interactions.

As noted in passing, class-IV modes involve the QCD penguin parameters a_4 and a_6 in the combination a_4+Ra_6, where $R>0$ for $B_s\to P_aP_b$, $R=0$ for P_aV_b and V_aV_b final states, and $R<0$ for $B_s\to V_aP_b$, where P_b or V_b is factorizable under the factorization assumption. Therefore, the decay rates of class-IV decays are expected to follow the pattern

$$\begin{aligned}\Gamma(B_s\to P_aP_b)&>\Gamma(B_s\to P_aV_b)\sim\Gamma(B_s\to V_aV_b)\\&>\Gamma(B_s\to V_aP_b),\end{aligned} \tag{47}$$

as a consequence of various possibilities of interference between the penguin terms characterized by the effective coefficients a_4 and a_6. From Tables III–IV, we see that

TABLE IV. Branching ratios (in units of 10^{-6}) averaged over CP-conjugate modes for charmless $\overline{B}_s \to VP$ decays. Predictions are for $k^2 = m_b^2/2$, $\eta = 0.34$, $\rho = 0.16$, and $N_c^{\rm eff}(LR) = 2,3,5,\infty$ with $N_c^{\rm eff}(LL)$ being fixed to be 2 in the first case and treated to be the same as $N_c^{\rm eff}(LR)$ in the second case. For decay modes involving the $B_s \to K^*$ or $B_s \to \phi$ transition, we use two different models for form factors: the BSW model [18] (the upper entry) and the light-cone sum rule approach [34] (the lower entry).

		$N_c^{\rm eff}(LL)=2$				$N_c^{\rm eff}(LL)=N_c^{\rm eff}(LR)$			
Decay	Class	2	3	5	∞	2	3	5	∞
$\overline{B}_s \to K^{*+}\pi^-$	I	4.30	4.30	4.30	4.30	4.30	4.79	5.20	5.84
		4.98	4.98	4.98	4.98	4.98	5.55	6.02	6.76
$\overline{B}_s \to K^+\rho^-$	I	17.2	17.2	17.2	17.2	17.2	19.2	20.8	23.4
$\overline{B}_s \to K^{0*}\pi^0$	II	0.14	0.14	0.14	0.15	0.14	0.01	0.02	0.23
		0.17	0.17	0.17	0.17	0.17	0.01	0.02	0.27
$\overline{B}_s \to K^0\rho^0$	II	0.55	0.55	0.55	0.55	0.55	0.06	0.13	1.00
$\overline{B}_s \to K^{*0}\eta'$	II,VI	0.10	0.14	0.18	0.26	0.10	0.04	0.04	0.15
		0.11	0.15	0.20	0.29	0.11	0.03	0.04	0.17
$\overline{B}_s \to K^{*0}\eta$	II,VI	0.18	0.18	0.18	0.17	0.18	0.13	0.17	0.39
		0.20	0.19	0.19	0.19	0.20	0.13	0.18	0.42
$\overline{B}_s \to K^0\omega$	II,VI	0.71	0.60	0.53	0.46	0.71	0.11	0.07	0.77
$\overline{B}_s \to K^{+*}K^-$	IV	0.68	0.78	0.87	1.01	0.68	0.75	0.80	0.88
		0.79	0.90	1.00	1.16	0.79	0.86	0.92	1.02
$\overline{B}_s \to K^{0*}\overline{K}^0$	IV	0.26	0.34	0.41	0.53	0.26	0.20	0.15	0.10
		0.31	0.40	0.48	0.62	0.31	0.23	0.18	0.11
$\overline{B}_s \to K^+K^{-*}$	IV	3.40	3.40	3.40	3.56	3.40	3.77	4.07	4.55
$\overline{B}_s \to K^0\overline{K}^{0*}$	IV	3.28	3.28	3.28	3.28	3.28	4.15	4.92	6.21
$\overline{B}_s \to \pi^0\phi$	V	0.18	0.18	0.18	0.17	0.18	0.22	0.27	0.40
		0.35	0.34	0.34	0.33	0.35	0.42	0.53	0.78
$\overline{B}_s \to \rho\eta'$	V	0.11	0.11	0.11	0.11	0.11	0.13	0.17	0.26
$\overline{B}_s \to \rho\eta$	V	0.11	0.11	0.11	0.11	0.11	0.14	0.18	0.26
$\overline{B}_s \to \omega\eta'$	V	0.79	0.18	0.01	0.31	0.79	0.004	0.36	2.52
$\overline{B}_s \to \omega\eta$	V	0.80	0.18	0.01	0.31	0.80	0.004	0.36	2.56
$\overline{B}_s \to \phi\eta'$	VI	1.06	1.18	1.28	1.45	1.06	0.27	0.22	1.11
		0.55	0.86	1.20	1.86	0.55	0.31	0.75	2.45
$\overline{B}_s \to \phi\eta$	VI	2.03	0.79	0.25	0.20	2.03	0.91	0.34	0.04
		1.43	0.41	0.15	0.69	1.43	0.58	0.19	0.09
$\overline{B}_s \to K^0\phi$	VI	0.002	0.01	0.04	0.13	0.002	0.03	0.10	0.29
		0.004	0.03	0.07	0.19	0.004	0.04	0.12	0.32

$$\Gamma(\overline{B}_s \to K^+K^-) > \Gamma(\overline{B}_s \to K^+K^{*-}) \gtrsim \Gamma(\overline{B}_s \to K^{*+}K^{*-}) > \Gamma(\overline{B}_s \to K^{+*}K^-),$$

$$\Gamma(\overline{B}_s \to K^0\overline{K}^0) > \Gamma(\overline{B}_s \to K^0\overline{K}^{*0}) \gtrsim \Gamma(\overline{B}_s \to K^{*0}\overline{K}^{*0}) > \Gamma(\overline{B}_s \to K^{*0}\overline{K}^0). \tag{48}$$

Note that the pattern $\Gamma(B \to P_aV_b) > \Gamma(B \to P_aP_b)$, which is often seen in tree-dominated decays, for example, $\Gamma(\overline{B}_s \to K^+\rho^-) > \Gamma(\overline{B}_s \to K^+\pi^-)$, occurs because of the larger spin phase space available to the former due to the existence of three different polarization states for the vector meson. On the contrary, the hierarchy (48) implies that the spin phase-space suppression of the penguin-dominated decay $B_s \to P_aP_b$ over $B_s \to P_aV_b$ or $B_s \to V_aP_b$ is overcome by the constructive interference between penguin amplitudes in the former. Recall that the coefficient R is obtained by applying equations of motion to the hadronic matrix elements of pseudoscalar densities induced by penguin operators. Hence, a test of the hierarchy shown in Eq. (48) is important for understanding the calculation of the penguin matrix element.[4]

[4]For a direct estimate of R using the perturbative QCD method rather than the equation of motion, see Ref. [23].

TABLE V. Same as Table IV except for $\overline{B}_s \to VV$ decays.

Decay	Class	$N_c^{\rm eff}(LL)=2$				$N_c^{\rm eff}(LL)=N_c^{\rm eff}(LR)$			
		2	3	5	∞	2	3	5	∞
$\overline{B}_s \to K^{+*}\rho^-$	I	12.5	12.5	12.5	12.5	12.5	13.9	15.0	16.9
		14.4	14.4	14.4	14.4	14.4	16.0	17.4	19.5
$\overline{B}_s \to K^{0*}\rho^0$	II	0.40	0.40	0.40	0.40	0.40	0.044	0.094	0.72
		0.46	0.46	0.46	0.46	0.46	0.051	0.11	0.84
$\overline{B}_s \to K^{0*}\omega$	II,VI	0.26	0.21	0.19	0.17	0.26	0.04	0.02	0.28
		0.30	0.25	0.22	0.19	0.30	0.044	0.031	0.32
$\overline{B}_s \to K^{+*}K^{-*}$	IV	2.53	2.53	2.53	2.53	2.53	2.80	3.03	3.38
		2.91	2.91	2.91	2.91	2.91	3.22	3.48	3.88
$\overline{B}_s \to K^{0*}\overline{K}^{0*}$	IV	2.44	2.44	2.44	2.44	2.44	3.09	3.66	4.62
		2.80	2.80	2.80	2.80	2.80	3.55	4.21	5.30
$\overline{B}_s \to \rho^0\phi$	V	0.17	0.18	0.18	0.18	0.17	0.22	0.28	0.41
		0.33	0.34	0.34	0.35	0.33	0.42	0.53	0.79
$\overline{B}_s \to \omega\phi$	V	0.65	0.15	0.01	0.25	0.65	0.004	0.30	2.09
		1.22	0.27	0.02	0.48	1.22	0.007	0.56	3.92
$\overline{B}_s \to K^{0*}\phi$	VI	0.007	0.049	0.10	0.19	0.007	0.13	0.28	0.57
		0.014	0.098	0.17	0.30	0.014	0.22	0.43	0.86
$\overline{B}_s \to \phi\phi$	VI	13.8	8.77	5.57	2.15	13.8	7.15	3.40	0.37
		25.1	15.9	10.1	3.91	25.1	13.0	6.18	0.68

TABLE VI. Fractions of nonelectroweak penguin contributions to the branching ratios of penguin-dominated two-body B_s decays, as defined by Eq. (43). Predictions are for $k^2=m_b^2/2$, $\eta=0.34$, $\rho=0.16$, and $N_c^{\rm eff}(LR)=2,3,5,\infty$ with $N_c^{\rm eff}(LL)$ being fixed to be 2 in the first case and treated to be the same as $N_c^{\rm eff}(LR)$ in the second case. We use the BSW model for form factors.

Decay	$N_c^{\rm eff}(LL)=2$				$N_c^{\rm eff}(LL)=N_c^{\rm eff}(LR)$			
	2	3	5	∞	2	3	5	∞
$\overline{B}_s \to \pi^0\eta'$	0.15	0.15	0.16	0.16	0.15	0.007	0.01	0.11
$\overline{B}_s \to \pi^0\eta$	0.15	0.15	0.16	0.16	0.15	0.007	0.01	0.11
$\overline{B}_s \to \pi^0\phi$	0.15	0.15	0.16	0.16	0.15	0.007	0.01	0.11
$\overline{B}_s \to \rho^0\eta'$	0.15	0.15	0.15	0.14	0.15	0.007	0.01	0.11
$\overline{B}_s \to \rho^0\eta$	0.15	0.15	0.15	0.14	0.15	0.007	0.01	0.11
$\overline{B}_s \to \omega\eta'$	0.78	0.57	1.63	1.43	0.78	0.79	1.42	1.16
$\overline{B}_s \to \omega\eta$	0.78	0.57	1.63	1.43	0.78	0.79	1.42	1.16
$\overline{B}_s \to \phi\eta'$	1.73	1.70	1.69	1.65	1.73	1.93	0.63	0.61
$\overline{B}_s \to \phi\eta$	1.71	2.16	3.01	0.39	1.71	2.00	2.58	2.81
$\overline{B}_s \to K^0\phi$	3.25	0.23	0.49	0.07	3.25	0.43	0.68	0.82
$\overline{B}_s \to K^{*+}K^{*-}$	0.87	0.87	0.87	0.87	0.87	0.94	1.00	1.08
$\overline{B}_s \to K^{*0}\overline{K}^{*0}$	1.08	1.08	1.08	1.08	1.08	1.03	1.00	0.96
$\overline{B}_s \to \rho^0\phi$	0.15	0.14	0.14	0.14	0.15	0.006	0.01	0.11
$\overline{B}_s \to \omega\phi$	0.78	0.57	1.63	1.43	0.78	0.79	1.42	1.16
$\overline{B}_s \to K^{0*}\phi$	3.82	0.55	0.76	0.86	3.82	0.75	0.84	0.87
$\overline{B}_s \to \phi\phi$	1.25	1.32	1.41	1.69	1.25	1.32	1.43	2.19

Among the 39 charmless two-body decay modes of the B_s meson, we find that only seven of them have branching ratios at the level of 10^{-5}:

$$\overline{B}_s \to K^+K^-,\ K^0\overline{K}^0,\ \eta\eta',\ \eta'\eta',\ K^+\rho^-,\ K^{+*}\rho^-,\ \phi\phi. \tag{49}$$

It is interesting to note that among the two-body rare decays of B^- and B_d, the class-VI decays $B^- \to \eta' K^-$ and $B_d \to \eta' K^0$ have the largest branching ratios [36]:

$$\mathcal{B}(B^\pm \to \eta' K^\pm) = (6.5^{+1.5}_{-1.4} \pm 0.9)\times 10^{-5},$$

$$\mathcal{B}(B_d \to \eta' K^0) = (4.7^{+2.7}_{-2.0} \pm 0.9)\times 10^{-5}. \tag{50}$$

The decay rate of $B^- \to \eta' K^-$ and $B_d \to \eta' K^0$ is large because they receive two different sets of penguin contributions proportional to $a_4 + Ra_6$ with $R>0$. By contrast, VP, VV modes in charm decays or bottom decays involving charmed mesons usually have larger branching ratios than the PP mode. Because of the strange quark content of the B_s, one will expect that the decay $B_s \to \eta\eta'$ or $B_s \to \eta'\eta'$, the B_s counterpart of $B_d \to \eta' K^0$, is the dominant two-body B_s decay. Our calculation indicates that while the branching ratio of $B_s \to \eta\eta'$ is large,

$$\mathcal{B}(B_s \to \eta\eta') \approx 2\times 10^{-5} \quad \text{for} \quad N_c^{\rm eff}(LL)=2,\ N_c^{\rm eff}(LR)=5, \tag{51}$$

it is only slightly larger than that of other decay modes listed in Eq. (49), see Tables III–V.

What is the role played by the intrinsic charm content of the η' to the hadronic charmless B_s decay? Just as the case of $B\to\eta' K$, $B_s\to\eta^{(\prime)}\eta'$ receives an internal W-emission contribution coming from the Cabibbo-allowed process $b\to c\bar{c}s$ followed by a conversion of the $c\bar{c}$ pair into the η' via gluon exchanges. Although the charm content of the η' is *a priori* expected to be small, its contribution is potentially important because the CKM mixing angle $V_{cb}V_{cs}^*$ is of the same order of magnitude as that of the penguin amplitude [see Eqs. (A10, A11)] and yet its effective coefficient a_2 is larger than the penguin coefficients by an order of magnitude. Since a_2 depends strongly on $N_c^{\rm eff}(LL)$ (see Table I), the contribution of $c\bar{c}\to\eta'$ is sensitive to the variation of $N_c^{\rm eff}(LL)$. It is easy to check that the η' charm content contributes in the same direction as the penguin terms at $1/N_c^{\rm eff}(LL)>0.28$ where $a_2>0$, while it contributes destructively at $1/N_c^{\rm eff}(LL)<0.28$ where a_2 becomes negative. In order to explain the abnormally large branching ratio of $B\to\eta' K$, an enhancement from the $c\bar{c}\to\eta'$ mechanism is certainly welcome in order to improve the discrepancy between theory and experiment. This provides another strong support for $N_c^{\rm eff}(LL)\approx 2$. Note that a similar mechanism explains the recent measurement of $B^-\to\eta_c K^-$ [37].

It turns out that the effect of the $c\bar{c}$ admixture in the η' is more important for $B_s\to\eta'\eta'$ than for $B_s\to\eta\eta'$. It is clear from Eq. (A1) that the destructive interference between $X_c^{(B_s\eta,\eta')}\propto f_{\eta'}^c F_0^{B_s\eta}$ and $X_c^{(B_s\eta',\eta)}\propto f_\eta^c F_0^{B_s\eta'}$ in the decay amplitude of $B_s\to\eta\eta'$, recalling that the form factors $F_0^{B_s\eta'}$ and $F_0^{B_s\eta}$ have opposite signs, renders the contribution of $c\bar{c}\to\eta'$ smaller for $B_s\to\eta\eta'$.

A very recent CLEO reanalysis of $B\to\eta' K$ using a data sample 80% larger than in previous studies yields the preliminary results [38]

$$\mathcal{B}(B^\pm\to\eta' K^\pm)=(7.4^{+0.8}_{-1.3}\pm 1.0)\times 10^{-5},$$
$$\mathcal{B}(B_d\to\eta' K^0)=(5.9^{+1.8}_{-1.6}\pm 0.9)\times 10^{-5}, \tag{52}$$

suggesting that the original measurements (50) were not an upward statistical fluctuation. This result certainly favors a slightly larger $f^c_{\eta^{(\prime)}}$ in magnitude than that used in Eq. (28). In fact, a more sophisticated theoretical calculation gives $f_{\eta'}^c=-(12.3\sim 18.4)$ MeV [29], which is consistent with all the known phenomenological constraints. This value of $f_{\eta'}^c$ will lead to an enhanced decay rate for $B\to\eta' K$. Numerically, we find that for $N_c^{\rm eff}(LL)=2, N_c^{\rm eff}(LR)=5$ and $f_{\eta'}^c=-15$ MeV,

$$\mathcal{B}(B_s\to\eta\eta')=2.2\times 10^{-5},\quad \mathcal{B}(B_s\to\eta'\eta')=1.8\times 10^{-5}, \tag{53}$$

to be compared with

$$\mathcal{B}(B_s\to\eta\eta')=1.8\times 10^{-5},\quad \mathcal{B}(B_s\to\eta'\eta')=1.2\times 10^{-5}, \tag{54}$$

in the absence of the intrinsic charm content of the η'.

Finally, we should point out the uncertainties associated with our predictions. Thus far, we have neglected W-annihilation, spacelike penguin diagrams, and final-state interactions; all of them are difficult to estimate. It is argued in Ref. [22] that these effects may play an essential role for our class-V and -VI decay modes. Other major sources of uncertainties come from the form factors and their q^2 dependence, the running quark masses at the scale m_b, the virtual gluon's momentum in the penguin diagram, and the values for the Wolfenstein parameters ρ and η.

IV. CONCLUSIONS

Using the next-to-leading oder QCD-corrected effective Hamiltonian, we have systematically studied hadronic charmless two-body decays of B_s mesons within the framework of generalized factorization. Nonfactorizable effects are parametrized in terms of $N_c^{\rm eff}(LL)$ and $N_c^{\rm eff}(LR)$, the effective numbers of colors arising from $(V-A)(V-A)$ and $(V-A)(V+A)$ four-quark operators, respectively. The branching ratios are calculated as a function of $N_c^{\rm eff}(LR)$ with two different considerations for $N_c^{\rm eff}(LL)$: (i) $N_c^{\rm eff}(LL)$ being fixed at the value of 2, and (ii) $N_c^{\rm eff}(LL)=N_c^{\rm eff}(LR)$. Depending on the sensitivity of the effective coefficients $a_i^{\rm eff}$ on $N_c^{\rm eff}$, we have classified the tree and penguin transitions into six different classes. Our results are

(1) The decays $\bar{B}_s\to\eta\pi$, $\eta'\pi$, $\eta\rho$, $\eta'\rho$, $\phi\pi$, $\phi\rho$ receive contributions only from the tree and electroweak penguin diagrams and are completely dominated by the latter. A measurement of them can be utilized to fix the effective electroweak penguin parameter a_9. For $N_c^{\rm eff}(LL)=2$, we found that electroweak penguin contributions account for 85% of their decay rates. Their branching ratios, though small [in the range of $(0.4-4.0)\times 10^{-7}$], could be accessible at hadron colliders with large b production.

(2) For class-V and -VI penguin-dominated modes $\bar{B}_s\to\omega\eta$, $\omega\eta'$, $\phi\eta$, $\omega\eta$, $K\phi$, $K^*\phi$, $\phi\phi$, electroweak penguin corrections, depending strongly on $N_c^{\rm eff}$, are as significant as QCD penguin effects and can even play a dominant role.

(3) Current experimental information on $B^-\to\omega\pi^-$ and $B^0\to\pi^+\pi^-$ favors a small $N_c^{\rm eff}(LL)$, that is, $N_c^{\rm eff}(LL)\approx 2$, which is also consistent with the nonfactorizable term extracted from $B\to(D,D^*)(\pi,\rho)$ decays, $N_c^{\rm eff}(B\to D\pi)\approx 2$. We have argued that the preferred value for the effective number of colors $N_c^{\rm eff}(LR)$ is $N_c^{\rm eff}(LR)\sim 5$.

(4) Because of various possibilities of interference between the penguin amplitudes governed by the QCD penguin parameters a_4 and a_6, the decay rates of class-IV decays follow the pattern: $\Gamma(\bar{B}_s\to P_aP_b)>\Gamma(\bar{B}_s\to P_aV_b)\sim\Gamma(\bar{B}_s\to V_aV_b)>\Gamma(\bar{B}_s\to V_aP_b)$, where $P_a=K^+$, $P_b=K^-$ or $P_a=K^0$, $P_b=\bar{K}^0$. A test of this hierarchy is important to probe the penguin mechanism.

(5) The decay $B\to\eta' K$ is known to have the largest branching ratios in the two-body hadronic charmless B^- and B_d decays. Its analogue in the B_s system, namely $B_s\to\eta\eta'$

has a branching ratio of order 2×10^{-5}, but it is only slightly larger than that of $\eta'\eta', K^{*+}\rho^-, K^+K^-, K^0\bar{K}^0$ decay modes, which have the branching ratios of order 10^{-5}.

(6) The recent CLEO reanalysis of $B\to\eta' K$ favors a slightly large decay constant $f^c_{\eta'}$. Using $f^c_{\eta'}=-15$ MeV, which is consistent with all the known theoretical and phenomenological constraints, we found that the intrinsic charm content of the η' is important for $B_s\to\eta'\eta'$, but less significant for $B_s\to\eta\eta'$.

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APPENDIX A: THE $\bar{B}_s\to PP$ DECAY AMPLITUDES

For $\bar{B}_s\to PP$ decays, we use $X^{(B_sP_1,P_2)}$ to denote the factorizable amplitude with the meson P_2 being factored out. Explicitly,

$$X^{(B_sP_1,P_2)}\equiv\langle P_2|(\bar{q}_2q_3)_{V-A}|0\rangle\langle P_1|(\bar{q}_1b)_{V-A}|\bar{B}_s\rangle = if_{P_2}(m^2_{B_s}-m^2_{P_1})F_0^{B_sP_1}(m^2_{P_2}). \quad \text{(A1)}$$

For a neutral P_1 with the quark content $N(\bar{q}q+\cdots)$, where N is a normalization constant,

$$X_q^{(B_sP_1,P_2)}\equiv\langle P_2|(\bar{q}q)_{V-A}|0\rangle\langle P_1|(\bar{q}_1b)_{V-A}|\bar{B}_s\rangle = if^q_{P_2}(m^2_{B_s}-m^2_{P_1})F_0^{B_sP_1}(m^2_{P_2}). \quad \text{(A2)}$$

As an example, the factorizable amplitudes $X^{(B_s\eta',K)}$ and $X_q^{(B_sK,\eta')}$ of the decay $\bar{B}_s\to\bar{K}^0\eta'$ read

$$X^{(B_s\eta',K)}=\langle\bar{K}^0|(\bar{s}d)_{V-A}|0\rangle\langle\eta'|(\bar{d}b)_{V-A}|\bar{B}_s\rangle = if_K(m^2_{B_s}-m^2_{\eta'})F_0^{B_s\eta'}(m^2_K),$$

$$X_q^{(B_sK,\eta')}=\langle\eta'|(\bar{q}q)_{V-A}|0\rangle\langle\bar{K}^0|(\bar{s}b)_{V-A}|\bar{B}_s\rangle = if^q_{\eta'}(m^2_{B_s}-m^2_K)F_0^{B_sK}(m^2_{\eta'}). \quad \text{(A3)}$$

For simplicity, W annihilation, spacelike penguins, and final-state interactions are not included in the decay amplitudes given below.

(1) $b\to d$ processes:

$$A(\bar{B}_s\to K^+\pi^-)=\frac{G_F}{\sqrt{2}}\left\{V_{ub}V^*_{ud}a_1-V_{tb}V^*_{td}\left[a_4+a_{10}+2(a_6+a_8)\frac{m^2_\pi}{(m_u+m_d)(m_b-m_u)}\right]\right\}X^{(B_sK^+,\pi^-)}, \quad \text{(A4)}$$

$$A(\bar{B}_s\to K^0\pi^0)=\frac{G_F}{\sqrt{2}}\left\{V_{ub}V^*_{ud}a_2-V_{tb}V^*_{td}\left[-a_4+\frac{3}{2}(-a_7+a_9)+\frac{1}{2}a_{10}-2\left(a_6-\frac{1}{2}a_8\right)\frac{m^2_\pi}{(m_d+m_d)(m_b-m_d)}\right]\right\}X_u^{(B_sK^0,\pi^0)}, \quad \text{(A5)}$$

$$\begin{aligned}A(\bar{B}_s\to K^0\eta^{(\prime)})=\frac{G_F}{\sqrt{2}}\Bigg(&V_{ub}V^*_{ud}a_2X_u^{(B_sK,\eta^{(\prime)})}+V_{cb}V^*_{cd}a_2X_c^{(B_sK,\eta^{(\prime)})}-V_{tb}V^*_{td}\Bigg\{\left[a_4-\frac{1}{2}a_{10}+2\left(a_6-\frac{1}{2}a_8\right)\frac{m^2_K}{(m_s+m_d)(m_b-m_d)}\right]\\&\times X^{(B_s\eta^{(\prime)},K)}+(a_3-a_5-a_7+a_9)X_u^{(B_sK,\eta^{(\prime)})}+\left(a_3-a_5+\frac{1}{2}a_7-\frac{1}{2}a_9\right)X_s^{(B_sK,\eta^{(\prime)})}+(a_3-a_5-a_7+a_9)\\&\times X_c^{(B_sK,\eta^{(\prime)})}+\left[a_3+a_4-a_5+\frac{1}{2}a_7-\frac{1}{2}a_9-\frac{1}{2}a_{10}+\left(a_6-\frac{1}{2}a_8\right)\frac{m^2_{\eta^{(\prime)}}}{m_s(m_b-m_s)}\left(\frac{f^s_{\eta^{(\prime)}}}{f^u_{\eta^{(\prime)}}}-1\right)r_{\eta'}\right]X_d^{(B_sK,\eta^{(\prime)})}\Bigg\}\Bigg).\end{aligned} \quad \text{(A6)}$$

(2) $b\to s$ processes:

$$A(\bar{B}_s\to K^+K^-)=\frac{G_F}{\sqrt{2}}\left\{V_{ub}V^*_{us}a_1-V_{tb}V^*_{ts}\left[a_4+a_{10}+2(a_6+a_8)\frac{m^2_K}{(m_u+m_s)(m_b-m_u)}\right]\right\}X^{(B_sK^+,K^-)}, \quad \text{(A7)}$$

$$A(\bar{B}_s\to\pi^0\eta^{(\prime)})=\frac{G_F}{\sqrt{2}}\left\{V_{ub}V^*_{us}a_2-V_{tb}V^*_{ts}\left[\frac{3}{2}(-a_7+a_9)\right]\right\}X_u^{(B_s\eta^{(\prime)},\pi^0)}, \quad \text{(A8)}$$

where

$$X_u^{(B_s\eta^{(\prime)},\pi^0)} \equiv \langle \pi^0|(\overline{u}u)_{V-A}|0\rangle\langle \eta^{(\prime)}|(\overline{s}b)_{V-A}|\overline{B}_s\rangle = i\frac{f_\pi}{\sqrt{2}}(m_{B_s}^2 - m_{\eta^{(\prime)}}^2)F_0^{B_s\eta^{(\prime)}}(m_\pi^2), \tag{A9}$$

$$\begin{aligned} A(\overline{B}_s\to\eta\eta') = \frac{G_F}{\sqrt{2}}\Bigg(& V_{ub}V_{us}^* a_2(X_u^{(B_s\eta,\eta')} + X_u^{(B_s\eta',\eta)}) + V_{cb}V_{cs}^* a_2(X_c^{(B_s\eta,\eta')} + X_c^{(B_s\eta',\eta)}) - V_{tb}V_{ts}^* \\ & \times\Bigg\{ \Bigg[a_3 + a_4 - a_5 + \frac{1}{2}a_7 - \frac{1}{2}a_9 - \frac{1}{2}a_{10} + \left(a_6 - \frac{1}{2}a_8\right)\frac{m_{\eta'}^2}{m_s(m_b-m_s)}\left(1 - \frac{f_{\eta'}^u}{f_{\eta'}^s}\right)\Bigg] X_s^{(B_s\eta,\eta')} \\ & + \left(2a_3 - 2a_5 - \frac{1}{2}a_7 + \frac{1}{2}a_9\right)X_u^{(B_s\eta,\eta')} + (a_3 - a_5 - a_7 + a_9)X_c^{(B_s\eta,\eta')} \\ & + \Bigg[a_3 + a_4 - a_5 + \frac{1}{2}a_7 - \frac{1}{2}a_9 - \frac{1}{2}a_{10} + \left(a_6 - \frac{1}{2}a_8\right)\frac{m_\eta^2}{m_s(m_b-m_s)}\left(1 - \frac{f_\eta^u}{f_\eta^s}\right)\Bigg] X_s^{(B_s\eta',\eta)} \\ & + \left(2a_3 - 2a_5 - \frac{1}{2}a_7 + \frac{1}{2}a_9\right)X_u^{(B_s\eta',\eta)} + (a_3 - a_5 - a_7 + a_9)X_c^{(B_s\eta',\eta)}\Bigg\}\Bigg), \end{aligned} \tag{A10}$$

$$\begin{aligned} A(\overline{B}_s\to\eta'\eta') = \frac{G_F}{\sqrt{2}}2\Bigg(& V_{ub}V_{us}^* a_2 X_u^{(B_s\eta',\eta')} + V_{cb}V_{cs}^* a_2 X_c^{(B_s\eta',\eta')} - V_{tb}V_{ts}^*\Bigg\{\Bigg[a_3 + a_4 - a_5 + \frac{1}{2}a_7 - \frac{1}{2}a_9 - \frac{1}{2}a_{10} \\ & + \left(a_6 - \frac{1}{2}a_8\right)\frac{m_{\eta'}^2}{m_s(m_b+m_s)}\left(1 - \frac{f_{\eta'}^u}{f_{\eta'}^s}\right)\Bigg] X_s^{(B_s\eta',\eta')} + \left(2a_3 - 2a_5 - \frac{1}{2}a_7 + \frac{1}{2}a_9\right)X_u^{(B_s\eta',\eta')} \\ & + (a_3 - a_5 - a_7 + a_9)X_c^{(B_s\eta',\eta')}\Bigg\}\Bigg). \end{aligned} \tag{A11}$$

The amplitude of $\overline{B}_s\to\eta\eta$ is obtained from $A(\overline{B}_s\to\eta'\eta')$ by the replacement $\eta'\to\eta$.

(3) Pure penguin process:

$$\begin{aligned} A(\overline{B}_s\to K^0\overline{K}^0) = \frac{G_F}{\sqrt{2}}\Bigg\{ & -V_{tb}V_{ts}^*\Bigg[a_4 - \frac{1}{2}a_{10} \\ & + 2\left(a_6 - \frac{1}{2}a_8\right)\frac{m_K^2}{(m_s+m_d)(m_b-m_d)}\Bigg]\Bigg\} \\ & \times X^{(B_sK^0,\overline{K}^0)}. \end{aligned} \tag{A12}$$

APPENDIX B: THE $\overline{B}_s\to VP$ DECAY AMPLITUDES

The factorizable amplitudes of $\overline{B}_s\to VP$ decays have the form

$$\begin{aligned} X^{(B_sP,V)} &\equiv \langle V|(\overline{q}_2 q_3)_{V-A}|0\rangle\langle P|(\overline{q}_1 b)_{V-A}|\overline{B}_s\rangle \\ &= 2f_V m_V F_1^{B_sP}(m_V^2)(\varepsilon\cdot p_{B_s}), \end{aligned}$$

$$\begin{aligned} X^{(B_sV,P)} &\equiv \langle P|(\overline{q}_2 q_3)_{V-A}|0\rangle\langle V|(\overline{q}_1 b)_{V-A}|\overline{B}_s\rangle \\ &= 2f_P m_V A_0^{B_sV}(m_P^2)(\varepsilon\cdot p_{B_s}). \end{aligned} \tag{B1}$$

For example, the factorizable terms $X^{(B_s\eta',K^*)}$ and $X_q^{(B_sK^*,\eta')}$ of $\overline{B}_s\to K^*\eta'$ decay are given by

$$\begin{aligned} X^{(B_s\eta',K^*)} &\equiv \langle K^{*0}|(\overline{s}u)_{V-A}|0\rangle\langle \eta'|(\overline{u}b)_{V-A}|\overline{B}_s\rangle \\ &= 2f_{K^*}m_{K^*}F_1^{B_s\eta'}(m_{K^*}^2)(\varepsilon\cdot p_{B_s}), \end{aligned}$$

$$\begin{aligned} X_q^{(B_sK^*,\eta')} &\equiv \langle \eta'|(\overline{q}q)_{V-A}|0\rangle\langle K^{*0}|(\overline{s}b)_{V-A}|\overline{B}_s\rangle \\ &= 2f_{\eta'}^q m_{K^*}A_0^{B_sK^*}(m_{\eta'}^2)(\varepsilon\cdot p_{B_s}). \end{aligned} \tag{B2}$$

(1) $b\to d$ processes:

$$A(\overline{B}_s \to K^{+*}\pi^-) = \frac{G_F}{\sqrt{2}}\Bigg\{ V_{ub}V_{ud}^* a_1 - V_{tb}V_{td}^* \times\Bigg[a_4 + a_{10} - 2(a_6+a_8) \times \frac{m_\pi^2}{(m_u+m_d)(m_b+m_u)}\Bigg]\Bigg\} \times X^{(B_s K^{+*},\pi^-)}, \tag{B3}$$

$$A(\overline{B}_s \to K^+\rho^-) = \frac{G_F}{\sqrt{2}}\{V_{ub}V_{ud}^* a_1 - V_{tb}V_{td}^*(a_4+a_{10})\} \times X^{(B_s K^+,\rho^-)}, \tag{B4}$$

$$A(\overline{B}_s \to K^{0*}\pi^0) = \frac{G_F}{\sqrt{2}}\Bigg\{ V_{ub}V_{ud}^* a_2 - V_{tb}V_{td}^*\Bigg[-a_4 - \frac{3}{2}a_7 + \frac{3}{2}a_9 + \frac{1}{2}a_{10} + 2\left(a_6 - \frac{1}{2}a_8\right) \times \frac{m_\pi^2}{2m_d(m_b+m_d)}\Bigg]\Bigg\} X_u^{(B_s K^{0*},\pi^0)}, \tag{B5}$$

$$A(\overline{B}_s \to K^0\rho^0) = \frac{G_F}{\sqrt{2}}\Bigg\{ V_{ub}V_{ud}^* a_2 - V_{tb}V_{td}^*\left(-a_4 + \frac{3}{2}a_7 + \frac{3}{2}a_9 + \frac{1}{2}a_{10}\right)\Bigg\} X_u^{(B_s K^0,\rho^0)}, \tag{B6}$$

$$A(\overline{B}_s \to K^0\omega) = \frac{G_F}{\sqrt{2}}\Bigg\{ V_{ub}V_{ud}^* a_2 - V_{tb}V_{td}^*\left(2a_3 + a_4 + 2a_5 + \frac{1}{2}a_7 + \frac{1}{2}a_9 - \frac{1}{2}a_{10}\right)\Bigg\} X_u^{(B_s K^0,\omega)}, \tag{B7}$$

$$A(\overline{B}_s \to \overline{K}^{*0}\eta^{(\prime)}) = \frac{G_F}{\sqrt{2}}\Bigg(V_{ub}V_{ud}^* a_2 X_u^{(B_s K^*,\eta')} + V_{cb}V_{cd}^* a_2 X_c^{(B_s K^*,\eta')} - V_{tb}V_{td}^*\Bigg\{ \left(a_4 - \frac{1}{2}a_{10}\right) X^{(B_s\eta',K^*)} + \Bigg[a_3 + a_4 - a_5 + \frac{1}{2}a_7 - \frac{1}{2}a_9 - \frac{1}{2}a_{10} - \left(a_6 - \frac{1}{2}a_8\right)\frac{m_{\eta'}^2}{m_s(m_b+m_s)}\left(\frac{f_{\eta'}^s}{f_{\eta'}^u} - 1\right) r_{\eta'}\Bigg] X_d^{(B_s K^*,\eta')} + (a_3 - a_5 - a_7 + a_9) X_u^{(B_s K^*,\eta^{(\prime)})} + \left(a_3 - a_5 + \frac{1}{2}a_7 - \frac{1}{2}a_9\right) X_s^{(B_s K^*,\eta^{(\prime)})} + (a_3 - a_5 - a_7 + a_9) X_c^{(B_s K^*,\eta^{(\prime)})}\Bigg\}\Bigg). \tag{B8}$$

(2) $b \to s$ processes:

$$A(\overline{B}_s \to K^+K^{-*}) = \frac{G_F}{\sqrt{2}}\{V_{ub}V_{us}^* a_1 - V_{tb}V_{ts}^*(a_4 + a_{10})\} X^{(B_s K^+,K^{-*})}, \tag{B9}$$

$$A(\overline{B}_s \to K^{+*}K^-) = \frac{G_F}{\sqrt{2}}\Bigg\{ V_{ub}V_{us}^* a_1 - V_{tb}V_{ts}^* \times\Bigg[a_4 + a_{10} - 2(a_6+a_8) \times \frac{m_K^2}{(m_s+m_u)(m_b+m_u)}\Bigg]\Bigg\} X^{(B_s K^{+*},K^-)}, \tag{B10}$$

$$A(\overline{B}_s \to \rho^0\eta^{(\prime)}) = \frac{G_F}{\sqrt{2}}\Bigg\{ V_{ub}V_{us}^* a_2 - V_{tb}V_{ts}^*\Bigg[\frac{3}{2}(a_7+a_9)\Bigg]\Bigg\} \times X_u^{(B_s\eta^{(\prime)},\rho^0)}, \tag{B11}$$

$$A(\overline{B}_s \to \omega\eta^{(\prime)}) = \frac{G_F}{\sqrt{2}}\Bigg\{ V_{ub}V_{us}^* a_2 - V_{tb}V_{ts}^*\Bigg[2(a_3+a_5) + \frac{1}{2}(a_7+a_9)\Bigg]\Bigg\} X_u^{(B_s\eta^{(\prime)},\omega)}, \tag{B12}$$

where

$$X_u{}^{(B_s\eta^{(\prime)},\omega)} \equiv \langle\omega|(\overline{u}u)_{V-A}|0\rangle\langle\eta^{(\prime)}|(\overline{s}b)_{V-A}|\overline{B}_s\rangle = \sqrt{2} f_\omega m_\omega F_1^{B\eta^{(\prime)}}(m_\omega^2)(\varepsilon\cdot p_{B_s}), \tag{B13}$$

$$A(\overline{B}_s\to\pi^0\phi)=\frac{G_F}{\sqrt{2}}\left\{V_{ub}V_{us}^*a_2-V_{tb}V_{ts}^*\times\left[\frac{3}{2}(-a_7+a_9)\right]\right\}X_u^{(B_s\phi,\pi^0)}, \quad \text{(B14)}$$

$$A(\overline{B}_s\to\phi\eta^{(\prime)})=\frac{G_F}{\sqrt{2}}\Bigg(V_{ub}V_{us}^*a_2X_u^{(B_s\phi,\eta^{(\prime)})}+V_{cb}V_{cs}^*a_2X_c^{(B_s\phi,\eta^{(\prime)})}-V_{tb}V_{ts}^*\times\Bigg\{\Bigg[a_3+a_4-a_5+\frac{1}{2}a_7-\frac{1}{2}a_9-\frac{1}{2}a_{10}-\left(a_6-\frac{1}{2}a_8\right)\frac{m^2_{\eta^{(\prime)}}}{m_s(m_b+m_s)}\left(1-\frac{f^u_{\eta^{(\prime)}}}{f^s_{\eta^{(\prime)}}}\right)\Bigg]\times X_s^{(B_s\phi,\eta^{(\prime)})}+\left(2a_3-2a_5-\frac{1}{2}a_7+\frac{1}{2}a_9\right)\times X_u^{(B_s\phi,\eta^{(\prime)})}+(a_3-a_5-a_7+a_9)\times X_c^{(B_s\phi,\eta^{(\prime)})}+\left(a_3+a_4+a_5-\frac{1}{2}a_7-\frac{1}{2}a_9-\frac{1}{2}a_{10}\right)X^{(B_s\eta^{(\prime)},\phi)}\Bigg\}\Bigg). \quad \text{(B15)}$$

(3) Pure penguin processes:

$$A(\overline{B}_s\to K^0\overline{K}^{0*})=-\frac{G_F}{\sqrt{2}}V_{tb}V_{ts}^*\left(a_4-\frac{1}{2}a_{10}\right)X^{(B_sK^0,\overline{K}^{0*})}, \quad \text{(B16)}$$

$$A(\overline{B}_s\to K^{0*}\overline{K}^0)=-\frac{G_F}{\sqrt{2}}V_{tb}V_{ts}^*\left[a_4-\frac{1}{2}a_{10}-2\left(a_6-\frac{1}{2}a_8\right)\times\frac{m_K^2}{(m_s+m_d)(m_b+m_d)}\right]X^{(B_sK^{0*},\overline{K}^0)}, \quad \text{(B17)}$$

$$A(\overline{B}_s\to K^0\phi)=-\frac{G_F}{\sqrt{2}}V_{tb}V_{td}^*\left\{\left[a_3+a_5-\frac{1}{2}(a_7+a_9)\right]\times X^{(B_sK^0,\phi)}+\left[a_4-\frac{1}{2}a_{10}-2\left(a_6-\frac{1}{2}a_8\right)\times\frac{m_K^2}{(m_s+m_d)(m_b+m_d)}\right]X^{(B_s\phi,K^0)}\right\}. \quad \text{(B18)}$$

APPENDIX C: THE $\overline{B}_s\to VV$ DECAY AMPLITUDES

The factorizable amplitude of $B_s\to VV$ decays has the form

$$X^{(B_sV_1,V_2)}=if_{V_2}m_{V_2}\Bigg[(\varepsilon_1^*\cdot\varepsilon_2^*)(m_{B_s}+m_{V_1})A_1^{B_sV_1}(m^2_{V_2})-(\varepsilon_1^*\cdot p_{B_s})(\varepsilon_2^*\cdot p_{B_s})\frac{2A_2^{B_sV_1}(m^2_{V_2})}{(m_{B_s}+m_{V_1})}+i\epsilon_{\mu\nu\alpha\beta}\varepsilon_2^{*\mu}\varepsilon_1^{*\nu}p^\alpha_{B_s}p_1^\beta\frac{2V^{B_sV_1}(m^2_{V_2})}{(m_{B_s}+m_{V_1})}\Bigg]. \quad \text{(C1)}$$

(1) $b\to d$ processes:

$$A(\overline{B}_s\to K^{+*}\rho^-)=\frac{G_F}{\sqrt{2}}\{V_{ub}V_{ud}^*a_1-V_{tb}V_{td}^*(a_4+a_{10})\}X^{(B_sK^{+*},\rho^-)}, \quad \text{(C2)}$$

$$A(\overline{B}_s\to K^{0*}\rho^0)=\frac{G_F}{\sqrt{2}}\left\{V_{ub}V_{ud}^*a_2-V_{tb}V_{td}^*\left(-a_4+\frac{3}{2}a_7+\frac{3}{2}a_9+\frac{1}{2}a_{10}\right)\right\}X_u^{(B_sK^{0*},\rho^0)}, \quad \text{(C3)}$$

$$A(\overline{B}_s\to K^{0*}\omega)=\frac{G_F}{\sqrt{2}}\left\{V_{ub}V_{ud}^*a_2-V_{tb}V_{td}^*\left(2a_3+a_4+2a_5+\frac{1}{2}a_7+\frac{1}{2}a_9-\frac{1}{2}a_{10}\right)\right\}X_u^{(B_sK^{0*},\omega)}. \quad \text{(C4)}$$

(2) $b\to s$ processes:

$$A(\overline{B}_s\to K^{+*}K^{-*})=\frac{G_F}{\sqrt{2}}\{V_{ub}V_{us}^*a_1-V_{tb}V_{ts}^*(a_4+a_{10})\}\times X^{(B_sK^{+*},K^{-*})}, \quad \text{(C5)}$$

$$A(\overline{B}_s\to\rho^0\phi)=\frac{G_F}{\sqrt{2}}\left\{V_{ub}V_{us}^*a_2-V_{tb}V_{ts}^*\left[\frac{3}{2}(a_7+a_9)\right]\right\}\times X_u^{(B_s\phi,\rho^0)}, \quad \text{(C6)}$$

$$A(\overline{B}_s\to\omega\phi)=\frac{G_F}{\sqrt{2}}\left\{V_{ub}V_{us}^*a_2-V_{tb}V_{ts}^*\left[2(a_3+a_5)+\frac{1}{2}(a_7+a_9)\right]\right\}X_u^{(B_s\phi,\omega)}. \tag{C7}$$

(3) Pure penguin processes:

$$A(\overline{B}_s\to K^{0*}\overline{K}^{0*})=-\frac{G_F}{\sqrt{2}}V_{tb}V_{ts}^*\left(a_4-\frac{1}{2}a_{10}\right)X^{(B_sK^{0*},\overline{K}^{0*})}, \tag{C8}$$

$$A(\overline{B}_s\to K^{0*}\phi)=-\frac{G_F}{\sqrt{2}}V_{tb}V_{td}^*\left\{\left[a_3+a_5-\frac{1}{2}(a_7+a_9)\right]\times X^{(B_sK^{0*},\phi)}+\left(a_4-\frac{1}{2}a_{10}\right)X^{(B_s\phi,K^{0*})}\right\}, \tag{C9}$$

$$A(\overline{B}_s\to\phi\phi)=-\frac{G_F}{\sqrt{2}}V_{tb}V_{ts}^*2\left[a_3+a_4+a_5-\frac{1}{2}(a_7+a_9+a_{10})\right]X^{(B_s\phi,\phi)}. \tag{C10}$$

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