Exploring $B_{d,s} \to KK$ Decays through Flavor Symmetries and QCD Factorization

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We present a new analysis of $B_{d,s} \to KK$ modes within the standard model (SM), relating them in a controlled way through $SU(3)$-flavor symmetry and QCD-improved factorization. We propose a set of sum rules for $B_{d,s} \to K^0 \bar{K}^0$ observables. We determine $B_s \to KK$ branching ratios and CP asymmetries as functions of $A_{\text{dir}}(B_d \to K^0 \bar{K}^0)$, pointing out a conflict between $BR(B_s \to K^+ K^-)$ in the SM and data. Finally, we predict the amount of U-spin breaking between $B_d \to \pi^+ \pi^-$ and $B_s \to K^+ K^-$.

B_s decays offer promising prospects in searches for new physics (NP). But to disentangle NP, it is essential to find new strategies to reduce hadronic uncertainties to obtain precise predictions within the standard model (SM). In particular, an ongoing effort has been devoted to $B_s \to KK$ decays within several approaches, mainly QCD factorization (QCDF) [1,2] (with its extension [3]) and flavor symmetries [4–7]. The former is a systematic expansion in $1/m_b$ but has difficulties with phenomenology due to power-suppressed hadronic effects, such as final-state interactions. The latter takes hadronic effects into account but may be affected by large corrections, up to 30% for $SU(3)$ relations. In this Letter, we combine the best of each method to derive SM relations between $B_d \to K^0 \bar{K}^0$ and $B_s \to KK$. We use data when available and exploit flavor symmetries and QCDF when they can be controlled.

This Letter is organized in the following way. First, we present two new SM relations for the $B_{d,s} \to K^0 \bar{K}^0$ decays that link the difference between tree and penguin contributions (a well-controlled quantity within QCDF) with observables measured in B experiments. Then we show that flavor symmetry yields interesting relations between hadronic parameters in $B_d \to K^0 \bar{K}^0$, $B_s \to K^0 \bar{K}^0$, and $B_s \to K^+ K^-$, providing a complementary strategy to $B_d \to \pi^+ \pi^-$ [4–7]. To exploit these relations, we propose to determine the $B_d \to K^0 \bar{K}^0$ hadronic parameters up to a twofold ambiguity from the branching ratio, the direct CP asymmetry and the tree-penguin difference. Third, we provide SM predictions for $B_s \to KK$ using this new strategy. Finally, we assess U-spin breaking between $B_s \to K^+ K^-$ and $B_d \to \pi^+ \pi^-$, of interest for both QCDF and flavor-symmetry approaches.

The SM amplitude for a B decaying into two mesons can be split into tree and penguin contributions [8]:

$$\bar{A} \equiv A(\bar{B}_q \to M\bar{M}) = \lambda_u^{(q)} T_M^{qC} + \lambda_c^{(q)} P_M^{qC}, \quad (1)$$

with C denoting the charge of the decay products, and the products of Cabibbo-Kobayashi-Maskawa (CKM) factors $\lambda_p^{(q)} = V_{pb} V_{pq}^*$. Using QCDF [1,2], one can perform a $1/m_b$ expansion of the amplitude, which gets two kinds of contributions [9]: a factorizable part which can be improved

within QCDF and a nonfactorizable one from $1/m_b$-suppressed corrections, which is delicate to evaluate.

The tree and penguin contributions in $\bar{B}_s \to K^+ K^-$ and $\bar{B}_s \to K^0 \bar{K}^0$ in QCDF are, respectively,

$$\hat{T}^{s\pm} = \bar{\alpha}_1 + \bar{\beta}_1 + \bar{\alpha}_4^u + \bar{\alpha}_{4EW}^u + \bar{\beta}_3^u$$
$$+ 2\bar{\beta}_4^u - \tfrac{1}{2}\bar{\beta}_{3EW}^u + \tfrac{1}{2}\bar{\beta}_{4EW}^u, \quad (2)$$

$$\hat{P}^{s\pm} = \bar{\alpha}_4^c + \bar{\alpha}_{4EW}^c + \bar{\beta}_3^c + 2\bar{\beta}_4^c - \tfrac{1}{2}\bar{\beta}_{3EW}^c + \tfrac{1}{2}\bar{\beta}_{4EW}^c, \quad (3)$$

$$\hat{T}^{s0} = \bar{\alpha}_4^u - \tfrac{1}{2}\bar{\alpha}_{4EW}^u + \bar{\beta}_3^u + 2\bar{\beta}_4^u - \tfrac{1}{2}\bar{\beta}_{3EW}^u - \bar{\beta}_{4EW}^u, \quad (4)$$

$$\hat{P}^{s0} = \bar{\alpha}_4^c - \tfrac{1}{2}\bar{\alpha}_{4EW}^c + \bar{\beta}_3^c + 2\bar{\beta}_4^c - \tfrac{1}{2}\bar{\beta}_{3EW}^c - \bar{\beta}_{4EW}^c, \quad (5)$$

where $\hat{P}^{sC} = P^{sC}/A_{KK}^s$, $\hat{T}^{sC} = T^{sC}/A_{KK}^s$, and $A_{KK}^q = M_{B_q}^2 F_0^{\bar{B}_q \to K}(0) f_K G_F/\sqrt{2}$. The superscripts identify the channel and the bar denotes quantities for decays with a spectator s quark. The tree and penguin contributions T^{d0} and P^{d0} for $\bar{B}_d \to K^0 \bar{K}^0$ have the same structure as Eqs. (4) and (5), with unbarred α's and β's recalling the different nature of the spectator d quark.

At next-to-leading order (NLO) in α_s, α's are linear combinations of vertex corrections, hard-spectator terms, and penguin contractions, whereas β's are sums of annihilation contributions. The weights of the various contributions are expressed in terms of α_s and Wilson coefficients [2]. The explicit form of $\bar{\alpha}_i^p - \alpha_i^p$ is shown in Sec. II. α's and β's contain the two most significant terms in the $1/m_b$ expansion: the LO terms, dominated by short distances, and the NLO terms in $1/m_b$ that include the potentially large long-distance corrections. The latter, parametrized in QCDF through quantities denoted X_H (in power corrections to the hard-scattering part of α_i) and X_A (in the annihilation parameters β_i), are singled out since they may upset the quick convergence of the $1/m_b$ expansion. The other $1/m_b$-suppressed contributions, dominated by short distances, are under control and small, i.e., leading to a $\mathcal{O}(5\%–10\%)$ error.

In this Letter, we show that comparing B_d and B_s decays into the same final states helps to cancel the potentially large long-distance $1/m_b$-suppressed effects ($X_{A,H}$), yielding improved SM predictions.

P H Y S I C A L   R E V I E W   L E T T E R S

I. Sum rules.—The difference $\Delta_d \equiv T^{d0} - P^{d0}$ plays a fundamental role here, since it is free from the troublesome NLO infrared divergence (modeled by $X_{A,H}$) that may be enhanced numerically by the chiral factor $r_\chi^K = 2m_K^2/m_b/m_s$ from twist-3 distribution amplitudes. Hard-scattering (X_H) and annihilation (X_A) terms occur in both penguin and tree contributions, but remarkably they cancel in the short-distance difference:

$$\Delta_d = A_{KK}^d[\alpha_4^u - \alpha_4^c + \beta_3^u - \beta_3^c + 2\beta_4^u - 2\beta_4^c]$$
$$= A_{KK}^d \alpha_s C_F C_1[\bar{G}(m_c^2/m_b^2) - \bar{G}(0)]/(4\pi N_c), \quad (6)$$

neglecting (small) electroweak contributions. The function $\bar{G} = G_K + r_\chi^K \hat{G}_K$ combines one-loop integrals from the penguin terms P_4 and P_6 defined in Sec 2.4 in Ref. [2]. The same cancellation of long-distance $1/m_b$ corrections happens for $\Delta_s \equiv T^{s0} - P^{s0}$. Taking into account the uncertainties coming from the QCDF inputs [2], we get $\Delta_d = (1.09 \pm 0.43) \times 10^{-7} + i(-3.02 \pm 0.97) \times 10^{-7} \text{ GeV}$ and $\Delta_s = (1.03 \pm 0.41) \times 10^{-7} + i(-2.85 \pm 0.93) \times 10^{-7} \text{ GeV}$.

These two theoretical quantities can be related to observables, namely, the corresponding branching ratio and coefficients of the time-dependent *CP* asymmetry:

$$\frac{\Gamma(B_d(t) \to K^0\bar{K}^0) - \Gamma(\bar{B}_d(t) \to K^0\bar{K}^0)}{\Gamma(B_d(t) \to K^0\bar{K}^0) + \Gamma(\bar{B}_d(t) \to K^0\bar{K}^0)}$$
$$= \frac{A_{\text{dir}}^{d0}\cos(\Delta Mt) + A_{\text{mix}}^{d0}\sin(\Delta Mt)}{\cosh(\Delta\Gamma_d t/2) - A_\Delta^{d0}\sinh(\Delta\Gamma_d t/2)}, \quad (7)$$

where we define [4] $A_{\text{dir}}^{d0} = (|A|^2 - |\bar{A}|^2)/(|A|^2 + |\bar{A}|^2)$, $A_\Delta^{d0} + iA_{\text{mix}}^{d0} = -(2e^{-i\phi_d}A^*\bar{A})/(|A|^2 + |\bar{A}|^2)$, and ϕ_d the phase of $B_d - \bar{B}_d$ mixing. A_Δ^{d0} is unlikely to be measured due to the small width difference $\Delta\Gamma_d$, but it can be obtained from $|A_\Delta^{d0}|^2 + |A_{\text{dir}}^{d0}|^2 + |A_{\text{mix}}^{d0}|^2 = 1$.

One can derive the following relation for $B_d \to K^0\bar{K}^0$:

$$|\Delta_d|^2 = \frac{BR^{d0}}{L_d}\{x_1 + [x_2\sin\phi_d - x_3\cos\phi_d]A_{\text{mix}}^{d0}$$
$$- [x_2\cos\phi_d + x_3\sin\phi_d]A_\Delta^{d0}\}, \quad (8)$$

where $L_d = \tau_d\sqrt{M_{Bd}^2 - 4M_K^2}/(32\pi M_{Bd}^2)$ and

$$x_1 = [|\lambda_c^{(d)}|^2 + |\lambda_u^{(d)}|^2 - 2|\lambda_c^{(d)}||\lambda_u^{(d)}|\cos\gamma]/n^2,$$
$$x_2 = -[|\lambda_c^{(d)}|^2 + |\lambda_u^{(d)}|^2\cos2\gamma - 2|\lambda_c^{(d)}||\lambda_u^{(d)}|\cos\gamma]/n^2,$$
$$x_3 = -[1 - \cos\gamma \times |\lambda_u^{(d)}|/|\lambda_c^{(d)}|]/n,$$

with $n = 2|\lambda_c^{(d)}||\lambda_u^{(d)}|\sin\gamma$. A similar relation between Δ_s and $B_s \to K^0\bar{K}^0$ observables is obtained by replacing $|\lambda_u^{(d)}| \to |\lambda_u^{(s)}|$, $|\lambda_c^{(d)}| \to -|\lambda_c^{(s)}|$, and $d \to s$ for all indices.

These sum rules can be used either as a SM consistency test between BR^{s0}, $|A_{\text{dir}}^{s0}|$, and A_{mix}^{s0} (and similarly for the $B_d^0 \to K^0\bar{K}^0$ observables), or as a way to extract the SM value of one observable (say, $|A_{\text{dir}}^{s0}|$) in terms of the two others (BR^{s0} and A_{mix}^{s0}) and Δ_s. These relations are free from the long-distance power-suppressed model-

dependent quantities X_A and X_H that are a main error source in the direct computation of A_{dir}^{s0} within QCDF.

II. Flavor symmetries and QCDF.—Using *U*-spin symmetry, we can relate the two penguin-mediated decays $\bar{B}_d \to K_0\bar{K}_0$ and $\bar{B}_s \to K_0\bar{K}_0$, as exemplified in Fig. 1 (see also Ref. [10] in relation to $B \to \pi\pi$). Let us stress that we work with the operators of the effective Hamiltonian: internal loops have already been integrated out to yield four-quark operators, so that the internal loop of the *u* penguin is not affected by *U*-spin rotations. *U*-spin breaking should be much smaller here than usual: it does not affect final-state interaction since both decays involve the same outgoing state, and it shows up mainly in power-suppressed effects. This is confirmed by QCDF:

$$P^{s0} = fP^{d0}[1 + (A_{KK}^d/P^{d0})\{\delta\alpha_4^c - \delta\alpha_{4EW}^c/2 + \delta\beta_3^c$$
$$+ 2\delta\beta_4^c - \delta\beta_{3EW}^c/2 - \delta\beta_{4EW}^c\}],$$
$$T^{s0} = fT^{d0}[1 + (A_{KK}^d/T^{d0})\{\delta\alpha_4^u - \delta\alpha_{4EW}^u/2 + \delta\beta_3^u$$
$$+ 2\delta\beta_4^u - \delta\beta_{3EW}^u/2 - \delta\beta_{4EW}^u\}], \quad (9)$$

where we define the *U*-spin breaking differences $\delta\alpha_i^p \equiv \bar{\alpha}_i^p - \alpha_i^p$ (idem for β). Apart from the factorizable ratio,

$$f = A_{KK}^s/A_{KK}^d = M_{B_s}^2 F_0^{\bar{B}_s \to K}(0)/[M_{B_d}^2 F_0^{\bar{B}_d \to K}(0)],$$

which should be computed on the lattice, *U*-spin breaking arises through $1/m_b$-suppressed contributions in which most long-distance contributions have cancelled out.

First, the hard-spectator scattering ($\delta\alpha$) probes the difference between B_d- and B_s-distribution amplitudes:

$$\delta\alpha_4^p = \alpha_s C_F C_3\pi/N_c^2 \times \delta\lambda_B \times [\langle\bar{x}\rangle_K^2 + r_\chi^K\langle x\rangle_K X_H^K],$$
$$\delta\lambda_B = B_{KK}^s M_{B_s}/(A_{KK}^s\lambda_{B_s}) - B_{KK}^d M_{B_d}/(A_{KK}^d\lambda_{B_d}).$$

$B_{KK}^q = f_{B_q}f_K^2 G_F/\sqrt{2}$, $\langle\bar{x}\rangle_K$ and M_{B_q}/λ_{B_q} are first and first inverse moments of K and B_q distribution amplitudes [2], respectively. $\delta\lambda_B$ is expected to be small, since the dynamics of the heavy-light meson in the limit $m_b \to \infty$ should vary little from B_d and B_s. Second, the annihilation contributions ($\delta\beta$) contain a *U*-spin breaking part when the gluon is emitted from the light quark in the $B_{d,s}$ meson (this effect from A_1^i and A_2^i defined in [2] is neglected in the QCDF model for annihilation terms).

Taking the hadronic parameters in [2], we obtain $|P^{s0}/(fP^{d0}) - 1| \lesssim 3\%$ and $|T^{s0}/(fT^{d0}) - 1| \lesssim 3\%$. These relations yield also the constraint $\Delta_s = f\Delta_d$ up to
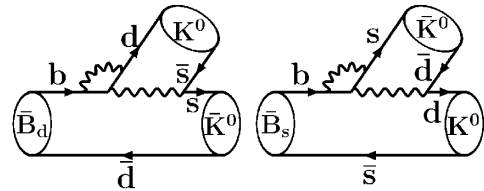

FIG. 1. Diagrams contributing to $\bar{B}_d \to K^0\bar{K}^0$ (left) and $\bar{B}_s \to K^0\bar{K}^0$ (right) related through *U*-spin transformations.

$1/m_b$-suppressed corrections, relating observables in $B_s \rightarrow K^0\bar{K}_0$ and $B_d \rightarrow K_0\bar{K}_0$.

Relations exist between $\bar{B}_d \rightarrow K_0\bar{K}_0$ and $\bar{B}_s \rightarrow K^+K^-$ as well. A combination of U-spin and isospin rotations leads from the penguin contribution in $\bar{B}_d \rightarrow K_0\bar{K}_0$ to that in $\bar{B}_s \rightarrow K_0\bar{K}_0$, then to $\bar{B}_s \rightarrow K^+K^-$, up to electroweak corrections (Fig. 1, up to replacing $d \rightarrow u$ in the right-hand diagram). There are no such relations between tree contributions, since $\bar{B}_s \rightarrow K^+K^-$ contains tree contributions which have no counterpart in the penguin-mediated decay $\bar{B}_d \rightarrow K_0\bar{K}_0$. This is seen in QCDF as well:

$$P^{s\pm} = fP^{d0}\left[1 + \frac{A^d_{KK}}{P^{d0}}\left\{\frac{3}{2}(\alpha^c_{4EW} + \beta^c_{4EW}) + \delta\alpha^c_4 + \delta\alpha^c_{4EW} + \delta\beta^c_3 + 2\delta\beta^c_4 - \frac{1}{2}(\delta\beta^c_{3EW} - \delta\beta^c_{4EW})\right\}\right],$$

$$\frac{T^{s\pm}}{A^s_{KK}\bar{\alpha}_1} = 1 + \frac{T^{d0}}{A^d_{KK}\bar{\alpha}_1} + \frac{1}{\bar{\alpha}_1}\left\{\bar{\beta}_1 + \frac{3}{2}(\alpha^u_{4EW} + \beta^u_{4EW}) + \delta\alpha^u_4 + \delta\alpha^u_{4EW} + \delta\beta^u_3 + 2\delta\beta^u_4 - \frac{1}{2}(\delta\beta^u_{3EW} - \delta\beta^u_{4EW})\right\}. \qquad (10)$$

Terms are ordered in decreasing size (in particular, curly brackets in $T^{s\pm}$ should be tiny). From QCDF, we obtain the following bounds: $|P^{s\pm}/(fP^{d0}) - 1| \lesssim 2\%$ and $|T^{s\pm}/(A^s_{KK}\bar{\alpha}_1) - 1 - T^{d0}/(A^d_{KK}\bar{\alpha}_1)| \lesssim 4\%$. The latter shows that flavor-symmetry breaking corrections are smaller than $T^{d0}/(A^d_{KK}\bar{\alpha}_1) = O(10\%)$. Fortunately, $T^{s\pm}$ is strongly CKM suppressed in $B_s \rightarrow K^+K^-$ so that the uncertainty on its QCDF determination will affect the branching ratio and CP asymmetries only marginally.

Finally, these relations between B_d and B_s hadronic parameters are affected by electroweak penguins, small in the SM but potentially enhanced by NP effects.

III. Hadronic parameters in $B_d \rightarrow K^0\bar{K}^0$.—The dynamics of $B_d \rightarrow K^0\bar{K}^0$ involves three hadronic real parameters (moduli of tree and penguin, and relative phase) which we can pin down through three observables: BR^{d0}, A^{d0}_{dir}, and A^{d0}_{mix}. Only $BR^{d0} = (0.96 \pm 0.25) \times 10^{-6}$ [11] has been measured. However, A^{d0}_{dir} should be observable fairly easily (for instance, $A^{d0}_{dir} = 0.19 \pm 0.06$ in QCDF), whereas the mixed asymmetry is likely small ($A^{d0}_{mix} = 0.05 \pm 0.05$ in QCDF). If only A^{d0}_{dir} becomes available, we have only 2 experimental constraints for 3 hadronic parameters. Then we may exploit a theoretically well-controlled QCDF constraint to get T^{d0} and P^{d0} from BR^{d0}, A^{d0}_{dir} and the QCDF value of $\Delta_d \equiv T^{d0} - P^{d0}$, free from infrared divergences and thus with little model dependence.

This system yields two constraints in the complex plane (x_P, y_P) for P^{d0}. First, the branching ratio defining $\rho_0^2 \equiv BR^{d0}/(2L_d)$ and the QCDF constraint on Δ_d yield a circular ring of center (x_C, y_C) and radius r:

$$x_C + iy_C = -\Delta_d(1 - \cos\gamma/R)/a,$$
$$r^2 = \rho_0^2/[a|\lambda^{(d)}_u|^2] - [\sin\gamma|\Delta_d|/(aR)]^2, \qquad (11)$$

with $a = 1 - 2\cos\gamma/R + 1/R^2$ and $R = |\lambda^{(d)}_u/\lambda^{(d)}_c|$.

The second constraint combines $\Delta_d = x_{\Delta_d} + iy_{\Delta_d}$ and the direct CP asymmetry A^{d0}_{dir} into a diagonal strip:

$$y_P x_{\Delta_d} = y_{\Delta_d} x_P - \rho_0^2 A^{d0}_{dir}/(2|\lambda^{(d)}_u \lambda^{(d)}_c|\sin\gamma). \qquad (12)$$

Numerically, only $|A^{d0}_{dir}| < 0.2$ is compatible with both constraints, which intersect in two points with opposite signs for ImP^{d0}, yielding two solutions for (P^{d0}, T^{d0}).

IV. SM predictions for $B_s \rightarrow KK$ decays.—Let us put the elements of our analysis together. From the measured value of the branching ratio for $B_d \rightarrow K^0\bar{K}^0$, and choosing a particular value of A^{d0}_{dir}, we get the penguin and tree contributions as explained in III. Then, the bounds in II yield the hadronic parameters in $B_s \rightarrow KK$ decays up to small uncertainties. To be more conservative, we actually stretch the bounds in II relating B_d and B_s hadronic parameters up to 5% in order to account for well-behaved short-distance $1/m_b$-suppressed corrections not yet included.

We obtain observables as functions of A^{d0}_{dir} in Table I. In the case of the branching ratios, we have split the error in two parts. The first uncertainty comes from the QCDF estimates of Δ_d and $\bar{\alpha}_1$, the theoretical constraints derived in II to relate B_d and B_s decays and the measurement of BR^{d0} (this experimental uncertainty dominates the others). The second error stems from (factorzable) U-spin breaking terms: $f = 0.94 \pm 0.2$ (cf. [2]).

Table I corresponds only to the solution of the constraints with Im$P^{d0} > 0$. But BR^{d0}, A^{d0}_{dir}, and Δ_d yield two different solutions for (T^{d0}, P^{d0}), and thus for $(T^{s\pm}, P^{s\pm})$. Only one solution is physical, whereas the other stems from the nonlinear nature of the constraints. We can use flavor-symmetry arguments to lift this ambiguity by exploiting a channel related to $B_s \rightarrow K^+K^-$ through U-spin, namely, $\bar{B}_d \rightarrow \pi^+\pi^-$ [4–7].

TABLE I. Observables for $\bar{B}_s \rightarrow K^0\bar{K}^0$ and $\bar{B}_s \rightarrow K^+K^-$ as functions of the direct asymmetry $A_{dir}(\bar{B}_d \rightarrow K^0\bar{K}^0)$ within the SM. We take $\lambda^{(d)}_u = 0.0038e^{-i\gamma}$, $\lambda^{(d)}_c = -0.0094$, $\lambda^{(s)}_u = 0.00088e^{-i\gamma}$, $\lambda^{(s)}_c = 0.04$, and $\gamma = 62°$, $\phi_d = 47°$, $\phi_s = -2°$ [12].

	$BR^{s0} \times 10^6$	$A^{s0}_{dir} \times 10^2$	$A^{s0}_{mix} \times 10^2$	$BR^{s\pm} \times 10^6$	$A^{s\pm}_{dir} \times 10^2$	$A^{s\pm}_{mix} \times 10^2$
$A^{d0}_{dir} = -0.2$	$18.4 \pm 6.5 \pm 3.6$	0.8 ± 0.3	-0.3 ± 0.8	$21.9 \pm 7.9 \pm 4.3$	24.3 ± 18.4	24.7 ± 15.5
$A^{d0}_{dir} = -0.1$	$18.2 \pm 6.4 \pm 3.6$	0.4 ± 0.3	-0.7 ± 0.7	$19.6 \pm 7.3 \pm 4.2$	35.7 ± 14.4	7.7 ± 15.7
$A^{d0}_{dir} = 0$	$18.1 \pm 6.3 \pm 3.6$	0 ± 0.3	-0.8 ± 0.7	$17.8 \pm 6.0 \pm 3.7$	37.0 ± 12.3	-9.3 ± 10.6
$A^{d0}_{dir} = 0.1$	$18.2 \pm 6.4 \pm 3.6$	-0.4 ± 0.3	-0.7 ± 0.7	$16.4 \pm 5.7 \pm 3.3$	29.7 ± 19.9	-26.3 ± 15.6
$A^{d0}_{dir} = 0.2$	$18.4 \pm 6.5 \pm 3.6$	-0.8 ± 0.3	-0.3 ± 0.8	$15.4 \pm 5.6 \pm 3.1$	6.8 ± 28.9	-40.2 ± 14.6

TABLE II. Hadronic parameters for $\bar{B}_s \to K^+K^-$ and U-spin breaking parameters $\mathcal{R}_\mathcal{C} = |T^{s\pm}/T^{d\pm}_{\pi\pi}|$ and $\xi = |P^{s\pm}/T^{s\pm}|/|P^{d\pm}_{\pi\pi}/T^{d\pm}_{\pi\pi}|$ relating $\bar{B}_s \to K^+K^-$ and $\bar{B}_d \to \pi^+\pi^-$. The last three columns correspond to $\bar{B}_s \to K^+K^-$ hadronic parameters from the second solution for $\bar{B}_d \to K^0\bar{K}^0$ tree and penguins (rejected due to a large U-spin violation).

	$|T^{s\pm}| \times 10^6$	$|P^{s\pm}/T^{s\pm}|$	$\arg(P^{s\pm}/T^{s\pm})$	$\mathcal{R}_\mathcal{C}$	ξ	$|T^{s\pm}|_{\text{rej}} \times 10^6$	$|P^{s\pm}/T^{s\pm}|_{\text{rej}}$	$\arg(P^{s\pm}/T^{s\pm})_{\text{rej}}$
$A^{d0}_{\text{dir}} = -0.2$	12.7 ± 2.8	0.09 ± 0.03	$(45 \pm 33)°$	2.3 ± 0.7	0.71 ± 0.24	13.1 ± 2.9	0.09 ± 0.03	$(-9 \pm 31)°$
$A^{d0}_{\text{dir}} = -0.1$	12.1 ± 2.7	0.10 ± 0.03	$(78 \pm 27)°$	2.2 ± 0.7	0.75 ± 0.27	12.8 ± 2.9	0.09 ± 0.03	$(-41 \pm 23)°$
$A^{d0}_{\text{dir}} = 0$	11.5 ± 2.6	0.10 ± 0.03	$(105 \pm 15)°$	2.1 ± 0.6	0.78 ± 0.31	12.3 ± 2.8	0.10 ± 0.03	$(-65 \pm 14)°$
$A^{d0}_{\text{dir}} = 0.1$	11.1 ± 2.6	0.11 ± 0.03	$(137 \pm 27)°$	2.0 ± 0.6	0.82 ± 0.35	11.8 ± 2.8	0.10 ± 0.03	$(-90 \pm 28)°$
$A^{d0}_{\text{dir}} = 0.2$	10.8 ± 2.6	0.11 ± 0.03	$(180 \pm 10)°$	2.0 ± 0.6	0.84 ± 0.35	11.2 ± 2.8	0.11 ± 0.03	$(-126 \pm 37)°$

First, we apply the method in [5,7] to the updated average [13] for $\bar{B}_d \to \pi^+\pi^-$: $BR = (5.0 \pm 0.4) \times 10^{-6}$, $A_{\text{dir}} = -0.33 \pm 0.11$, and $A_{\text{mix}} = 0.49 \pm 0.12$. In this way, we obtain the tree and penguin contributions $|T^{d\pm}_{\pi\pi}| = (5.48 \pm 0.42) \times 10^{-6}$, $|P^{d\pm}_{\pi\pi}/T^{d\pm}_{\pi\pi}| = 0.13 \pm 0.05$, and $\arg(P^{d\pm}_{\pi\pi}/T^{d\pm}_{\pi\pi}) = (131 \pm 18)°$, which agree well with their $\bar{B}_s \to K^+K^-$ counterparts as confirmed by the first columns of Table II (for $A^{d0}_{\text{dir}} > 0$), corresponding to the solution with $\text{Im}P^{d0} > 0$. We get also the U-spin breaking parameters $\mathcal{R}_\mathcal{C}$ and ξ. The last columns give the hadronic parameters for the second solution ($\text{Im}P^{d0} < 0$), to be discarded: U-spin would be strongly broken by the phase of the ratio, and we get $A^{s\pm}_{\text{dir}} < 0$ contrary to U-spin predictions from $\bar{B}_d \to \pi^+\pi^-$ [7,8]. Thus the twofold ambiguity can be lifted based on U-spin and data on $\bar{B}_d \to \pi^+\pi^-$.

Table I shows the sign anticorrelation between $A^{s\pm}_{\text{mix}}$ and A^{d0}_{dir}. U-spin arguments applied to $\bar{B}_d \to \pi^+\pi^-$ data indicate $A^{s\pm}_{\text{mix}} \lesssim 0$ [7], and thus $A^{d0}_{\text{dir}} \gtrsim 0$. Another interesting issue is $BR^{s\pm}$, whose determination is improved compared to the U-spin extraction from $\bar{B}_d \to \pi^+\pi^-$ [5,7]. Its value is a bit low compared to CDF data [14]:

$$BR^{s\pm}|_{\text{th}} \times 10^6 = 20 \pm 8 \pm 4 \pm 2 \quad [\text{our SM result}],$$

$$BR^{s\pm}|_{\text{exp1}} \times 10^6 = 33 \pm 9 \quad [\bar{B}_d \to \pi^+K^- \text{ ratio}],$$

$$BR^{s\pm}|_{\text{exp2}} \times 10^6 = 42 \pm 15 \quad [\bar{B}_d \to \pi^+\pi^- \text{ ratio}].$$

Our SM result is obtained by averaging over the whole range of A^{d0}_{dir}, although a less conservative restriction to $A^{d0}_{\text{dir}} > 0$ would yield slightly lower central value. The first uncertainty comes from $BR(B_d \to K^0\bar{K}^0)$ and A^{d0}_{dir}, accounting for long-distance $1/m_b$ corrections. The second one comes from the factorizable ratio f. When relating B_d and B_s hadronic parameters in II, the error bars have been stretched to account for $1/m_b$-suppressed contributions that are not enhanced and thus not included in $X_{A,H}$. In addition to this stretching, we give a rough estimate of the same nonenhanced $1/m_b$-suppressed terms through the last error quoted for our SM result. Within these fairly conservative errors, the data suggest a departure from the SM, to be further checked experimentally.

V. Conclusions.—We have combined experimental data, flavor symmetries, and QCDF to propose sum rules for $B_{d,s} \to K^0\bar{K}^0$ observables and to give SM constraints on $B_s \to K\bar{K}$ in a controlled way. We have correlated $B_s \to$ $K\bar{K}$ observables to the direct $B_d \to K^0\bar{K}^0$ CP asymmetry and predicted the U-spin breaking parameter ξ. The main errors on our results in Table I come from the U-spin breaking ratio f (computable on the lattice) and from the experimental value of $BR(B_d \to K^0\bar{K}^0)$. If sizeable NP effects occur, the SM correlations between B_d and B_s decays exploited here should be broken, leading to departure from our predictions. Indeed we have pointed out a potential conflict between the SM prediction for $BR^{s\pm}$ and experimental data. The ideas developed here could be applied to other nonleptonic B decays.

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